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Is concentrated vorticity that important? [1]

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ABSTRACT. – The main purpose of this paper is to bring about a better balance between views of the exaggerated importance of concentrated vorticity, on one hand, and the underestimated role of three kinds of regions *other* than concentrated vorticity: *i* – 'structureless' background, *ii* – regions of strong vorticity/strain (self) interaction and strong enstrophy generation, and *iii* – regions with negative enstrophy production, on the other hand, with the emphasis on the latter. Results of experiments on turbulent grid flow and DNS of decaying turbulent flow in a periodic 'box' at the same Reynolds number ($\mathrm{Re}_\lambda \approx 75$) are used in order to demonstrate that *all* these regions are strongly non-Gaussian, dynamically significant and possess structure. It is argued that due to the strong nonlocality of turbulence in physical space all the four regions are in continuous interaction and are strongly correlated.

Thus the answer to the question posed in the title is that – though important – regions of concentrated vorticity are not as important as is commonly believed. © Elsevier, Paris

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1. Introductory notes and motivation

1.1. General

One of the prominent and distinctive features of turbulent flows, of the utmost dynamical significance, is the build up of *odd* moments (Antonia *et al.* 1997, van Atta and Antonia 1980, Betchov 1976, Brachet *et al.* 1992, Hill 1998, Katul *et al.* 1995, Kerr 1985, Kolmogorov 1941, Lindborg 1996, Orszag 1977, Taylor 1938b, Tsinober *et al.* 1992, 1997, Vainshtein 1997), which among other things means phase and geometrical coherency, *i.e.* structure. The non-Gaussian characteristics both at large and small scales are exhibited not only by nonzero odd moments but also in the departure of even moments from their Gaussian values. Thus both the large and small scales differ essentially from the Gaussian *indicating* that both possess structure. The term *indicating* is used, since non-Gaussian behaviour is only a necessary condition for structure (whatever this means). Until recently very little was known of what this structure is or what the structures look like. The structure in question is the so called fine structure and not that which is promoted by various external factors/constraints such as boundaries, mean shear, centrifugal forces (rotation), buoyancy, magnetic fields, etc., which usually act as an organizing factor, favoring the formation of coherent structures of different kinds (quasi-two-dimensional, helical, hairpins, etc.). These are as a rule large scale features which depend on the particular features of a given flow and thus are not universal. It is noteworthy that the statement that turbulence has structure is in a sense trivial: to say that turbulent flow is 'completely random' would define turbulence out of existence (Tritton 1988, p. 295) -

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[1] This overview is based in part on lectures delivered by the author in *École Normale Supérieure, Université Paris VII* in May 1995 and in *Laboratoire Modélisation en Mécanique, Université Paris VI* in February 1996 and on the latest work performed since then (Tsinober and Shtilman 1998, Tsinober *et al.* 1997, 1998).

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after all turbulent flows seem to obey the Navier-Stokes equations. On the other hand a pure Gaussian velocity field has some structure too (*see Fig.* 3 in She *et al.* 1990).

Since the first DNS simulations (Siggia, 1981) a number of other computations (Ashurst *et al.* 1987, Hosokawa and Yamamoto 1989, Kerr 1985, Kida 1993, Jimenez *et al.* 1993, Jimenez and Wray 1994ab, Passot *et al.* 1995, Ruetsch and Maxey 1991,1992, She *et al.* 1990, 1991, Tanaka and Kida 1993, Vincent and Meneguzzi 1991, Yamamoto and Hosokawa 1988, Yeung *et al.* 1995) and also laboratory visualisation experiments (Bonn *et al.* 1993, Cadot *et al.* 1995, Douady *et al.* 1991, Jacob and Savas 1995, Schwarz 1990, Villermaux *et al.* 1995) have been performed which demonstrate clearly that fluid-dynamical turbulence which is 'homogeneous' and 'isotropic' has 'structure', *i.e.* contains a variety of strongly localized events. The primary evidence is related to spatial localization of subregions with large enstrophy (i.e. intense vorticity) which are organized in long, thin tubes-filaments-worms, though there is nothing unique in vorticity in the sense that many other quantities exibit distinct localized structure as well.

All this prompted the belief that essential aspects of the fine structure of turbulence and its dynamics may be adequately represented by a random distribution of strained vortical objects (sheets, filaments, tubes-worms-sinews, eigenmodes) and enhanced interest in studying the dynamics and stability of such (individual) structures (Andreotti 1997, Kambe 1986, Le Dizes *et al.* 1996, Moffatt *et al.* 1994, Rossi and Le Dizès 1997 and references therein). This idea goes back to Townsend (Townsend, 1951; *see* also Batchelor 1953, pp. 159 -161) and it has been developed to a high level of sophistication (Hatakeyama and Kambe 1997, Gilbert 1993, Kambe and Hosokawa 1995, Lundgren 1982, 1993, Pullin and Saffman 1995, Saffman and Pullin 1996, Segel 1995 and references therein). The main feature and shortcoming of these objects is that they possess single-component vorticity, zero curvature of vortex lines and consequently such objects lack the 'genuine' nonlinearity — self-amplification: they are stretched by the strain which is decoupled from them.

It is also commonly believed that *most* of the structure of turbulence is associated with and is due to various strongly localized intense events/structures, e.g. mostly regions of concentrated vorticity so that ' *turbulent flow is dominated by vortex tubes of small cross-section and bounded eccentricity*' (Chorin 1994, p. 95) and that these events are mainly responsible for the phenomenon of intermittency (Belin *et al.* 1996a, Frisch 1995, Katul *et al.* 1994, Nelkin 1995 and references therein).

1.2. On the role of concentrated vorticity

However, recent numerical experiments clearly demonstrate that the concentrated vorticity (tubes-filaments-worms) is not as important as it was thought before and '*do not seem to play a special role in the overall dynamics of turbulent flows*' and that '*the effect of removing the worms is small,...*' (Jimenez et al. 1993). In other words, the 'worms' are more the *consequence rather than the dominating factor of the turbulence dynamics.*

The belief that regions of concentrated vorticity are mostly responsible for the phenomenon of intermittency seems to be an exaggeration too. There is some evidence (Belin *et al.* 1996a) that the vortex filaments are in part responsible for the intermittency 'corrections' in the scaling laws of the higher order velocity structure functions and of dissipation in particular experiments both numerical and laboratory (Belin *et al.* 1996a, Douady *et al.* 1991, Cadot *et al.* 1995a, Jimenez *et al.* 1993, Jimenez and Wray 1994, Vincent and Meneguzzi 1991; *see* also Abry *et al.* 1994, Passot *et al.* 1992). The word 'particular' was added for several reasons. First, since the flow in the laboratory experiments of Belin *et al.* 1996a, Douady *et al.* 1991, Cadot *et al.* 1995a and in DNS by Brachet 1991 (*see* also Boratav and Pelz 1997) possess regions of very strong mean shear it is quite possible that many of the observed filaments originate in these regions. Indeed, in the experiments with oscillating grids by Villermaux *et al.* 1995, in which no mean shear was present, the frequency of appearance of such filaments was essentially smaller. The apparent transition and structural change above some Re_λ in the flow properties

(Belin *et al.* 1996a, Chabaud *et al.* 1994, Emsellem *et al.* 1997) may be due to changes in the stability of the *large scale* properties of this flow, which exert considerable influence on its small scale properties (Labbe *et al.* 1996a,b, Mordant *et al.* 1997). Nevertheless, this change in the events, which are thought to be responsible for the intermittency effects, resulted in no observable change in the scaling exponents of the structure functions (Arneodo *et al.* 1996, Belin *et al.* 1996b). Similarly, the structures observed in numerical experiments may be - at least in part - the result of the strong constraints imposed on the flow by the periodic boundary conditions, which form a kind of large scale forcing. [2] Third, vortex models of fine scale turbulence based on ensembles consisting of vortex filaments only (including the so called Burgers/Kambe/Lundgren strained vortices) may not exhibit any intermittency corrections whatsoever and conform with the Kolmogorov scaling as in Chorin 1994, Segel 1995, in contradiction with what one would expect if the vortex filaments were (mainly) responsible for the intermittency corrections in the scaling exponents. Recent results from DNS by Boratav and Pelz 1997 indicate that concentrated vorticity does not contribute significantly to the anomalous scaling of longitudinal structure functions, but do contribute to that of transverse structure functions (for more on this matter *see* Tsinober 1998). Fourth, anomalous scaling very similar to that in fluid dynamical turbulence is observed in many models of turbulence, which are based on qualitatively different premises/assumptions, and with few exceptions have no direct bearing to NSE, e.g. in pure temporal systems without any spatial structure, such as the so called GOY model (Benzi *et al.* 1996, Dombre and Gilson 1998, Gledzer *et al.* 1996, Kadanoff *et al.* 1995, Leveque and She 1997). Therefore, it is somewhat misleading to call them models of turbulence - the original name 'systems of hydrodynamical type' proposed by (Obukhov 1969) seems to be more appropriate. Some anomalous scaling was reported recently even for low order ($p < 2$) structure functions (Cao *et al.* 1996a, b), which cannot 'feel' strongly localized events like vortex tubes-filaments-worms. Strong intermittency exists in linear problems with so called multiplicative noise such as for a passive scalar or vector advected by *any* (*i.e.* not necessarily real, e.g. pure Gaussian and 'structureless') random velocity field (Chertkov *et al.* 1996, Kraichnan 1995, Kraichnan and Kimura 1994, Pumir *et al.* 1997, Reyl *et al.*, 1997, Rogachevski and Kleorin 1997, Shraiman an Siggia 1996, Vergassola 1996, Zel'dovich *et al.* 1978 and references therein). Finally, it has been observed in experiments on counter-rotating disks with polymer additives that the dissipation remained the *same* as with water, but the number of filaments was strongly reduced (Douady and Couder 1993, Bonn *et al.* 1993, Cadot *et al.* 1997).

All this (for more *see* the main text) is a clear indication that – though important – the regions of concentrated vorticity are not as important as is commonly believed.

We concentrate our attention on three kinds of dynamically important regions in turbulent flows other than concentrated vorticity regions:

- – Background - structureless random sea?

Partially as a consequence of the accepted view on the role of concentrated vorticity, regions with 'weak' excitation are thought to be structureless, *i.e.* comprise a kind of highly uncorrelated 'random sea' of limited dynamical significance, in which are embedded the intense and strongly localized structures, which dominate the turbulent flow. Moreover, the 'random sea', *i.e.* regions with 'weak' excitation, is frequently considered to be nearly Gaussian (She 1991, She *et al.* 1991 and references therein). However, visual observations (Bonn *et al.* 1993, Douady *et al.* 1991, Douady and Couder 1993, Cadot et al. 1995a,b) clearly indicate that the 'random sea', in which are embedded the strongest filaments, contains a variety of much smaller filaments

[2] Indeed the correlation coefficient between two values of *any* quantity at such opposing boundaries, i.e. the points separated at *maximal distance* in the flow domain is precisely equal to unity and close to unity for the points in the proximity of such boundaries. On the contrary in any *real* flow the correlation coefficient becomes very small for points separated by a distance of the order of and larger than the integral scale of turbulent flow.

of different scales. This is an indication that the 'random sea' with 'weak' excitation (vorticity, strain, etc.) is not that random and possesses distinct structure. In this context it is worth noting that the 'random sea', the 'grassroot' contributes most to the total dissipation, enstrophy, and enstrophy generation, *i.e.* the 'random sea' is of major dynamical importance.

- – Regions of strong (local) vorticity/strain interaction.

There exist regions (intense and weak — both structured and dynamicaly active) other than concentrated vorticity regions, which in some respects are dynamicaly more important than those of concentrated vorticity (Tsinober *et al.* 1992, 1995b, 1997, Vincent and Meneguzzi 1994), e.g. they contribute most to the enstrophy generation. These regions are associated mainly with largest strain (large Λ_1) rather than enstrophy (Ruetsch and Maxey 1991), a tendency to alignment between $\boldsymbol{\omega}$ and the eigenvector $\boldsymbol{\lambda}_1$ associated with the largest eigenvalue Λ_1 of the rate of strain tensor s_{ij} (Tsinober *et al.* 1995b, 1997), and fairly large curvature of vorticity lines (Tsinober *et al.* 1998).

- – About a third of the volume in turbulent flows is occupied by regions with vortex compressing, *i.e.* negative enstrophy generation, alignment between $\boldsymbol{\omega}$ and $\boldsymbol{\lambda}_3$, large magnitude of Λ_3, which to a large extent (but not only) is related to vortex folding and tilting. This part of turbulent flows plays an important role too, e.g. in generating a considerable part of the curvature of vortex lines along with the other part produced in the regions of largest enstrophy generation via self-induction (Tsinober *et al.* 1998).

One of purposes of this paper is to bring more balance between the views of the exaggerated importance of concentrated vorticity, on one hand, and the underestimated role of the three regions just mentioned, on the other hand, with the emphasis on the latter.

2. Geometrical statistics

The above as well as other issues related to structure of turbulence can be effectively addressed *quantitatively* via *geometrical statistics* which is one of the main themes of this presentation (Constantin 1994, Corrsin 1972, Tsinober 1996, Tsinober *et al.* 1995a,b, 1997, 1998 and references therein). The qualitative individual observations of the structure of turbulence – being extremely useful – are inherently limited as comparred to statistical information. Geometrical statistics allow us to address, at least in part, the quantitative statistical aspects of the problem.

The widely known example of the very great importance of geometrical relations in turbulence is the qualitative difference between the dynamics of 3D and 2D turbulence. This is seen immediately from the equations for vorticity ω_i and enstrophy ω^2

$$D_t\omega_i = \omega_j s_{ij} + \nu\nabla^2\omega_i, \qquad D_t(\omega^2/2) = \omega_i\omega_j s_{ij} + \nu\omega_i\nabla^2\omega_i, \tag{1,2}$$

or alternatively an equation for the magnitude of vorticity ω and the unit vector $\tilde{\boldsymbol{\omega}} = \boldsymbol{\omega}/\omega$ $(\varpi_i = \cos(\boldsymbol{\omega}, \boldsymbol{\lambda}_i))$ defining the direction of vorticity (Constantin 1994)

$$D_t\omega = \alpha\omega + vt, \quad D_t\varpi_i = s_{ij}\varpi_j - \alpha\varpi_i + vt, \tag{3,4}$$

where $D_t \equiv \partial/\partial t + u_k(\partial/\partial x_k)$, vt stands for viscous terms and $\alpha = \frac{\omega_i\omega_j s_{ij}}{\omega^2} = \Lambda_i\varpi_i^2 \equiv \Lambda_i\cos^2(\boldsymbol{\omega}, \boldsymbol{\lambda}_i)$ is the rate of (inviscid) enstrophy generation or rate of change of vorticity magnitude, which depends on the strain and the mutual orientation of $\boldsymbol{\omega}$ and the eigenbasis $\boldsymbol{\lambda}_i$ of the rate of strain tensor only and does not depend explicitly on the magnitude of $\boldsymbol{\omega}$. In the following text the eigenvalues of the rate of strain tensor are denoted as Λ_i and $\Lambda_1 > \Lambda_2 > \Lambda_3$ while $\Lambda_1 + \Lambda_2 + \Lambda_3 = 0$ due to incompressibility, so that $\Lambda_1 > 0$ and $\Lambda_3 < 0$. It is known from experiments — both numerical (Ashurst *et. al* 1987) and laboratory (Tsinober *et. al* 1992) — that $\langle\Lambda_2\rangle > 0$, *i.e.* the PDF of Λ_2 is positively skewed. The corresponding eigenvectors are denoted as $\boldsymbol{\lambda}_i$.

The nonlinear terms $\omega_j s_{ij}$ and $\omega_i\omega_j s_{ij}$ are known to be responsible for the so called vortex stretching and enstrophy generation. In other words the essential nonlinear dynamics of 3D-turbulence is contained in the interaction between vorticity $\boldsymbol{\omega}$ and the rate of strain tensor s_{ij}. Both $\omega_j s_{ij}$ and $\omega_i\omega_j s_{ij}$ vanish identically for 2-D flows. It is well known (Betchov 1976, Brachet *et al.* 1992, Taylor 1938, Tsinober *et al.* 1992, 1995a,b) that in 3-D turbulence $\langle\omega_i\omega_j s_{ij}\rangle$ is an essentially positive quantity - the PDF of $\omega_i\omega_j s_{ij}$ is strongly positively skewed. Moreover, *all* $\int_{V_L}\omega_i\omega_j s_{ij} dV_L$ over volumes of the order of integral scale are essentially positive too (*Fig.* 1). This reflects one of the most basic *specific* properties of three-dimensional turbulent flows - the prevalence of the vortex stretching process: the enstrophy generation $\omega_i\omega_j s_{ij}$ is an outstanding nonzero *odd* moment of the utmost dynamical importance in turbulence. It is noteworthy that about a third of the flow volume is occupied by regions with negative $\omega_i\omega_j s_{ij} < 0$. These regions play an important role in the dynamics of turbulence. For example, these regions make a positive contribution to the magnitude of the vortex stretching vector $W_i \equiv \omega_j s_{ij}$ in equation (1). Indeed, $W^2 = \omega_i^2\Lambda_i^2\cos^2(\boldsymbol{\omega},\boldsymbol{\lambda}_i)$ and is large for large $\omega_3^2\Lambda_3^2\cos^2(\boldsymbol{\omega},\boldsymbol{\lambda}_3)$ for which the enstrophy generation $\sigma \equiv \omega_i\omega_j s_{ij} = \omega_i^2\Lambda_i\cos^2(\boldsymbol{\omega},\boldsymbol{\lambda}_i)$ is negative. Similarly, enstrophy generation can be small, whereas W can be large (*see* Table I). This implies that in both cases large curvature of vortex lines can be generated. For more and on other aspects *see* sections 7, 8 and Tsinober 1998, Tsinober *et al.* 1998.

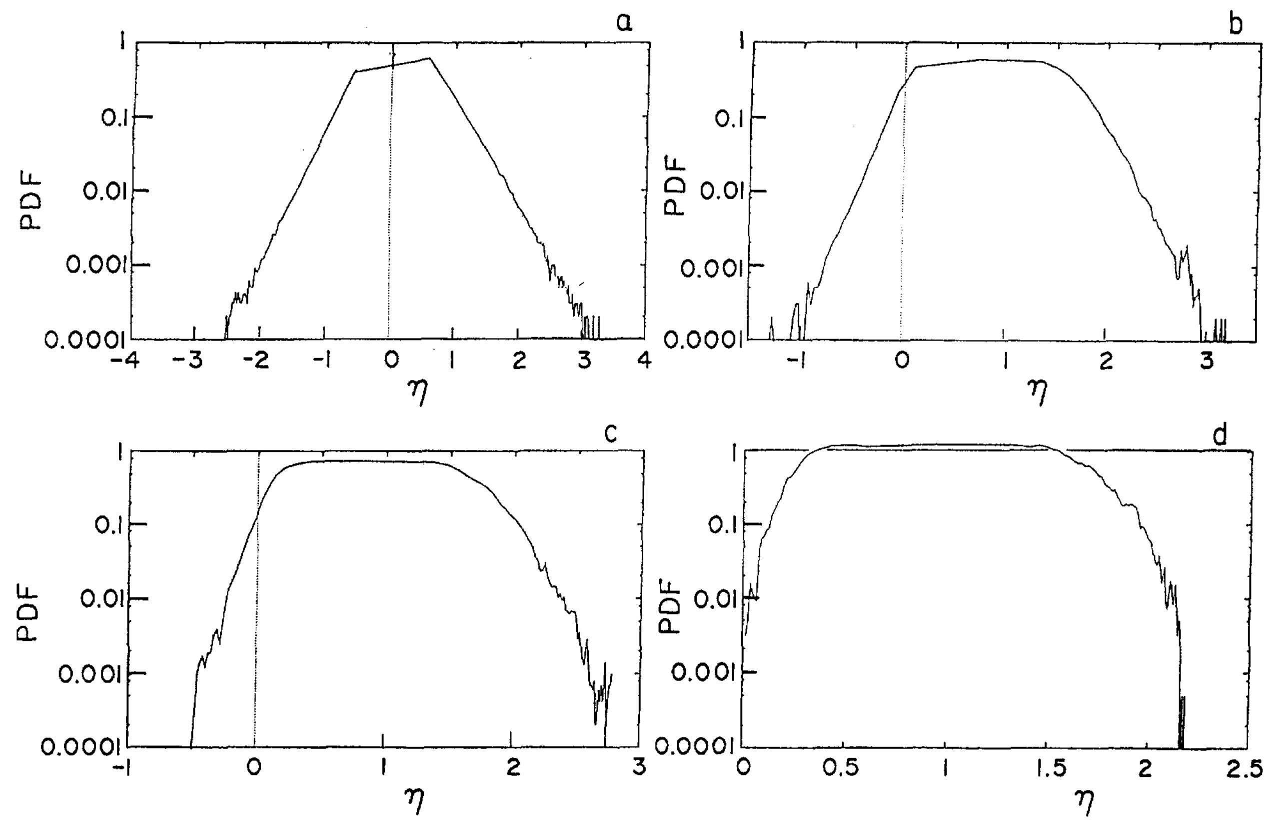

Fig. 1. – PDFs of enstrophy generation $\sigma \equiv \omega_i\omega_j s_{ij}$ normalized by its mean $\langle\sigma\rangle$ in grid turbulence experiments at $\mathrm{Re}_\lambda \approx 75$, Tsinober *et al* 1992, 1995b. For convenience the quantity $x = \{\sigma/\langle\sigma\rangle - \sigma_0\}^{1/3}$ is shown in *a*) – *d*). It has the same sign as σ for any σ_0. *a*) - 'pointwise': the scale of the probe is $\approx 4\eta$, where η is the Kolmogorov scale; *b*) – *d*) running average over: b) - $L \approx 80\eta \approx 0.1\mathcal{L}$, $\sigma_0 = 0.4$, c) $L \approx 200\eta \approx 0.25\mathcal{L}$, $\sigma_0 = 0.7$, d)$L \approx 1000\eta \approx 1.25\mathcal{L}$, $\sigma_0 = 0.9$; $\mathcal{L}$ is the integral scale. Note that on the integral scale *all* the 'events' have positive entrophy generation only.

TABLE I. – Contribution to the total mean of the magnitude of vortex stretcing vector $\langle W^2 \rangle \equiv \langle \omega^2 \Lambda_i^2 \cos^2(\omega, \lambda_i) \rangle$ and its rate $\langle W^2/\omega^2 \rangle \equiv \langle \Lambda_i^2 \cos^2(\omega, \lambda_i) \rangle$ from the terms corresponding to the eigenvalues Λ_i of the rate of strain tensor s_{ij}. DNS, $Re_\lambda \approx 75$.

$\langle \omega^2 \Lambda_1^2 \cos^2(\omega, \lambda_1) \rangle$	$\langle \omega^2 \Lambda_2^2 \cos^2(\omega, \lambda_2) \rangle$	$\langle \omega^2 \Lambda_3^2 \cos^2(\omega, \lambda_3) \rangle$
0.53	0.15	0.32
$\langle \Lambda_1^2 \cos^2(\omega, \lambda_1) \rangle$	$\langle \Lambda_2^2 \cos^2(\omega, \lambda_2) \rangle$	$\langle \Lambda_3^2 \cos^2(\omega, \lambda_3) \rangle$
0.51	0.11	0.38

Quantities like $\omega_i \omega_j s_{ij}$ are geometrical invariants, e.g. they remain invariant under the full group of rotations in contradistinction with other *noninvariant* combinations of velocity derivatives. For this reason the geometrical invariants are believed to be the most appropriate for studying physical processes in turbulent flows, their structure and universal properties. 'Surrogates' of the type $(\partial u_1/\partial x_1)^n$ reperesent adequately only the *means* of the 'true' quantities such as mean disspation. Other properties (spectral, fractal, scaling, etc.) of the surrogates and of the 'true' quantities (invariants) are generally different (Tsinober *et al.* 1992, Tsinober 1995, 1996, 1998 and references therein; for some recent results on such differences *see* Hosokawa and Ode 1996). Similarly there exist qualitative differences between the flow regions dominated by strain and those by vorticity (e.g. Blackburn *et al.* 1996, Boratav and Pelz 1997, Chen *et al.* 1997, Hunt *et al.* 1988, She *et al.* 1991, Zeldovich *et al.* 1990). A new aspect of such a difference is addressed in section 6.

Along with the enstrophy generation $\omega_i \omega_j s_{ij}$ and another third order invariant $s_{ij} s_{ik} s_{kj}$ six fourth order invariants are of importance $\mathcal{I}_1 = s_{ik} s_{kj} s_{il} s_{lj} \equiv s^4$; $\mathcal{I}_2 = \omega^2 s^2 \equiv \omega_i \omega_i s_{kj} s_{kj}$; $\mathcal{I}_3 = \omega_i s_{ij} \omega_k s_{ik} \equiv W^2$; $\mathcal{I}_4 = \omega^4$; $\mathcal{I}_5 = \omega_i \omega_j \frac{\partial^2 p}{\partial x_i \partial x_j}$; $\mathcal{I}_6 = s_{ik} s_{kj} \frac{\partial^2 p}{\partial x_i \partial x_j}$ (Tsinober 1995). The two last quantities reflect the nonlocal *dynamical* effects due to pressure (*see* section 8).

Among other things vortex stretching is responsible for the enhanced dissipation in turbulent flows (Taylor 1938) and it is the primary mechanism for formation of structures. So far no theoretical arguments have been given in favor of the positiveness of $\langle \omega_i \omega_j s_{ij} \rangle$. The argument that the reason is the (approximate) balance between the enstrophy generation and enstrophy dissipation is misleading and puts the consequences before the causes, since it is known that for Euler equations enstrophy generation increases with time very quickly – apparently without limit (Brachet *et al.* 1992, Chorin 1994, Fernandez *et al.* 1995, Grauer and Sideris 1995, Greene and Boratav 1997, Kerr 1993, 1996, Ohkitani 1997). Another rather common view that the prevalence of vortex stretching is due to the predominance of stretching of material lines is - at best - true in part only, since, there exist several qualitative differences between the two processes (*see* Appendix).

A usual phenomenological argument (e.g. Frisch 1995) results in the following estimate $\alpha \sim \omega$ (or s, $s^2 \equiv s_{ij} s_{ij}$), whereas in reality it is only $\alpha \sim \omega^{n_\omega}$ with $n_\omega \sim 0.3 \div 0.5$ in slots of ω (*see Fig.* 2 and *Fig.* 17 in Jimenez *et al.* 1993). However, $\alpha \sim s^{n_s}$ with $n_s \sim 1$ in slots of s (*Fig.* 2). This shows the importance of taking into account the mutual orientation of vorticity $\boldsymbol{\omega}$ and the eigenframe $\boldsymbol{\lambda}_i$ $(i = 1, 2, 3)$ of the rate of strain tensor s_{ij}, since $\alpha = \frac{\omega_i \omega_j s_{ij}}{\omega^2} = \Lambda_i \cos^2(\boldsymbol{\omega}, \boldsymbol{\lambda}_i)$. In other words the essential dynamics of 3D-turbulence contained in the interaction between vorticity $\boldsymbol{\omega}$ and the rate of strain tensor s_{ij} depends strongly not only on the magnitude of vorticity and strain but also on the geometry of the field of velocity derivatives, in particular on the mutual orientation of vorticity $\boldsymbol{\omega}$ and the eigenframe $\boldsymbol{\omega}_i$ of the rate of strain tensor s_{ij}. One the simplest means to characterizing this geometrical aspect of turbulence dynamics quantitatively is to look at the alignments between vorticity and strain, e.g. the PDFs of the cosine of the angle between vorticity $\boldsymbol{\omega}$ and the eigenvectors $\boldsymbol{\lambda}_i$.

2.1. ALIGNMENTS

Various alignments comprise important simple *geometrical* characteristics and manifestations of the *dynamics and structure* of turbulence. For example, there is a distinct qualitative difference between the PDFs of cos

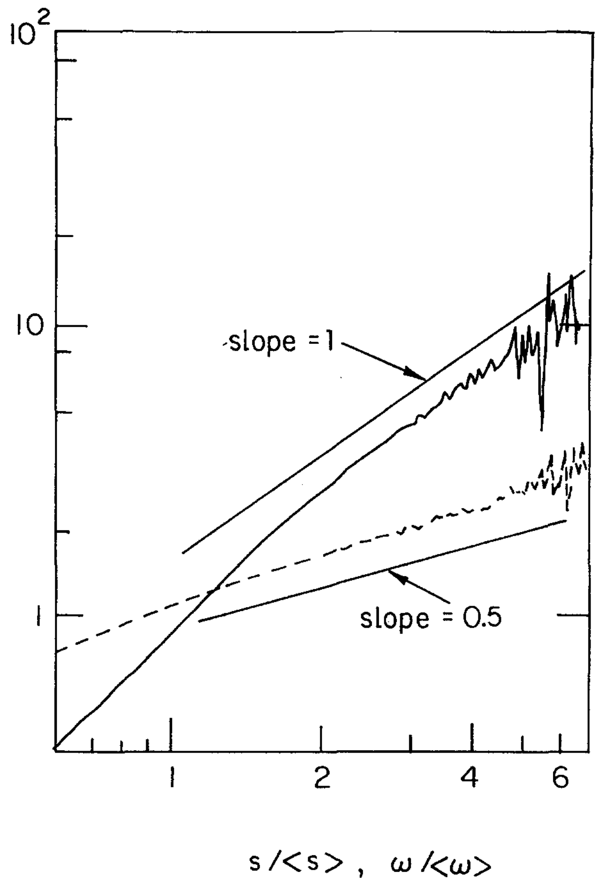

Fig. 2. – Rate of enstrophy generation α in *slots* of ω and s in DNS of NSE for $\mathrm{Re}_\lambda \approx 75$; ——— - $x = \langle\alpha|_s\rangle/\langle\alpha\rangle$, - - - - - $x = \langle\alpha|_\omega\rangle/\langle\alpha\rangle$. One can see that the rate of enstrophy generation in *slots* of ω grow much slower than in *slots* of s. This behaviour cannot be predicted by phenomenological arguments.

$(\boldsymbol{\omega}, \boldsymbol{\lambda}_i)$ for a real turbulent flow and a random Gaussian velocity field. In the latter *all* these PDFs are precisely flat! (Shtilman *et al.* 1993). Another important example is the strict alignment between vorticity and the vortex stretching vector $W_i \equiv \omega_j s_{ij}$. In real turbulent flows it is strongly asymmetric (Gibbon and Heritage 1997, Shtilman *et al.* 1993, Tsinober *et al.* 1992, 1995a,b, 1997, 1998) in full conformity with the prevalence of vortex stretching over vortex compressing, *i.e.* positive value of $\langle\omega_i\omega_j s_{ij}\rangle \equiv \langle\boldsymbol{\omega}\cdot\mathbf{W}\rangle$, whereas it is symmetric for a random Gaussian field (Shtilman *et al.* 1993). Thus the very existence of alignments such as mentioned above points to the presence of internal organization of flow at various scales, i.e. alignments belong to the rare *quantitative statistical* manifestation of the existence of structure in turbulence. They are the simplest representative of a much broader class of geometrical statistics in turbulent flows. It is noteworthy that while the above mentioned (and some other) alignments are intimately related to the dynamics of turbulent flows, there are alignments which are mostly of kinematic nature, e.g. alignment between the Lamb vector $\boldsymbol{\omega}\times\mathbf{u}$ and its potential part (pressure gradient), the alignment between velocity and the eigenvectors of rate of strain tensor and some others (Shtilman *et al.* 1993, Tsinober 1990, Tsinober *et al.* 1995, Tsinober 1996).

Alignments by their very definition are suitable for events of any magnitude, since they do not contain the amplitude of the quantities involved. Finally, alignments are invariant in the sense that they are independent of the system of reference and therefore, along with other invariant quantities are the most appropriate in studying physical processes generally and in particular for characterization of the structural nature of turbulent flows.

3. Turbulence background – stuctureless random sea?

The above mentioned properties of alignments make it possible to answer a number of questions on turbulence structure in a simple and reliable way, such as for example, whether the regions with 'small' excitation do possess structure. The answer is definitely yes. It appears that contrary to the common view the so called 'background' is strongly non-Gaussian, is not dynamically passive and is not structureless (Figures 3 – 6).

Note the tendency for alignment between $\boldsymbol{\omega}$ and $\boldsymbol{\lambda}_2$, especially for strong vorticity (*Fig.* 3). This alignment was discovered by Ashurst *et al.* 1987 and was quite unexpected, since the accepted paradigm was that vorticity should align with the eigenvector $\boldsymbol{\lambda}_1$ corresponding to the maximal eigenvalue of the rate of strain tensor, though alignment between $\boldsymbol{\omega}$ and $\boldsymbol{\lambda}_2$ was conjectured as early as 1956 (Betchov 1956, *see* also Siggia 1981 and Kerr 1985). This expectation was based on an analogy with the alignments of material elements which indeed tend to align with $\boldsymbol{\lambda}_1$ (*see* Appendix). Though in the background the tendency for alignment between $\boldsymbol{\omega}$ and $\boldsymbol{\lambda}_2$ is weaker, it is still significant especially taking into account that the background (say $\omega^2 < \langle\omega^2\rangle$) is occupying about 70 of the flow volume (*cf.* with the volume occupied by strong vorticity, say $\omega^2 > 3\langle\omega^2\rangle$, which occupies only about 6% of the flow volume). This significance is stressed in Figure 4, showing the normalized enstrophy generation $\omega_i\omega_j S_{ij}|\omega|^{-1}|W|^{-1} = \cos(\boldsymbol{\omega}, \mathbf{W})$, which is just the cosine of the angle between vorticity and the vortex stretching vector $\mathbf{W}$ ($W_i = \omega_i s_{ij}$). Note the strong asymmetry of its PDF for the background $\omega^2 < \langle\omega^2\rangle$, which is almost the same as for the whole field. This asymmetry remains significant even for $\omega^2 < 0.1\langle\omega^2\rangle$, as well as for *both* $\omega^2 < 0.1\langle\omega^2\rangle$ and $s^2 < 0.1\langle s^2\rangle$ (not shown), and becomes *stronger* for

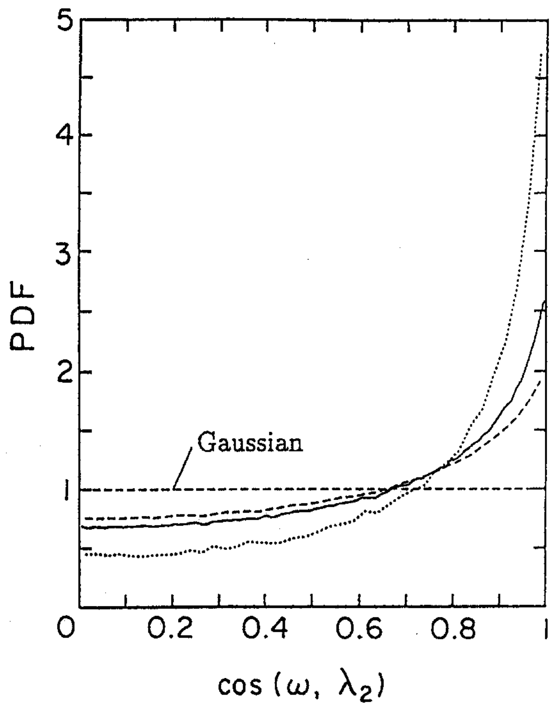

Fig. 3. – PDFs of the cosine of the angle between vorticity and the intermediate eigenvector of the rate of strain tensor for grid turbulence $Re_\lambda \approx 75$; —— whole field, - - - - $\omega^2 < \langle\omega^2\rangle$, · · · · · $\omega^2 > 3\langle\omega^2\rangle$, Tsinober *et al.* 1992. Note that for weak enstrophy ($\omega^2 < \langle\omega^2\rangle$) - contrary to common beliefs - this PDF is considerably different from that for a Gaussian field.

$\omega^2 < \langle\omega^2\rangle$ and $\cos(\boldsymbol{\omega}, \boldsymbol{\lambda}_2) > 0.9$. The above points are emphasized in Figures 5 and 6, in which the joint PDFs of $\cos(\boldsymbol{\omega}, \boldsymbol{\lambda}_2)$ and ω^2, and $\cos(\boldsymbol{\omega}, \mathbf{W})$ and ω^2, are shown. It is seen that the maxima of these PDFs are located at *weakest* enstrophy and *strongest* alignment between $\boldsymbol{\omega}$ and $\boldsymbol{\lambda}_2$, and $\boldsymbol{\omega}$ and $\mathbf{W}$. The same is true for a variety of joint PDFs of other quantities (Tsinober *et al.* 1995b, 1997), showing clearly that the background is strongly non-Gaussian, not structureless and not inactive. Note that - as explained in detail in Tsinober *et al.* 1995b, 1997 - this does not contradict the mostly known result on the tendency to alignment between $\boldsymbol{\omega}$ and $\boldsymbol{\lambda}_2$ in regions of concentrated vorticity: the regions with such an alignment are an order of magnitude larger than those with concentrated vorticity only.

It should be emphasized that the preferential alignment of vorticity $\boldsymbol{\omega}$ with the eigenvector $\boldsymbol{\lambda}_2$ associated with the intermediate eigenvalue Λ_2 of the rate of strain tensor is only in apparent contradiction with the view that the largest eigenvalue Λ_1 should play an essential role in the dynamics of turbulent flows. Indeed, as shown in Tsinober *et al.* 1995b, 1997, this is really the case, e.g. most of the enstrophy generation is really associated with Λ_1 (*see* sections 4 and 6). An additional aspect is that the preferential alignment between $\boldsymbol{\omega}$ and $\boldsymbol{\lambda}_2$ occurs on the same scale, whereas simultaneosly vorticity $\boldsymbol{\omega}$ tends to align with the principal eignevector $\boldsymbol{\lambda}_1^f$ of the filtered large scale (in space/time) strain (Kevlahan and Hunt 1997, Porter *et al.* 1998).

All the results shown above were obtained in turbulent flow past a grid at rather low Reynolds number ($\mathrm{Re}_\lambda \approx 75$) and very similar results were obtained in DNS simulations of decaying turbulence at the same Reynolds number – a detailed comparison of both is given in Tsinober *et al.* 1997. However, a number of

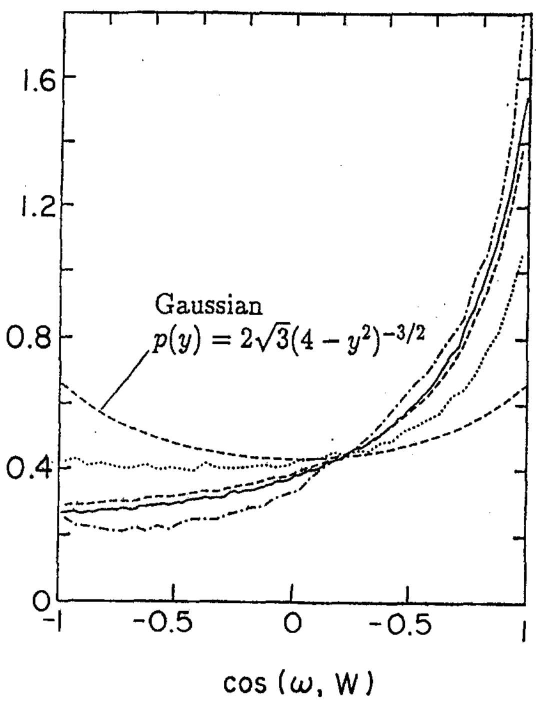

Fig. 4. – PDFs of the cosine of the angle between vorticity and the vortex stretching vector $\mathbf{W}, W_i = \omega_i s_{ij}$, for grid turbulence $\mathrm{Re}_\lambda \approx 75$; —— whole field, - - - - $\omega^2 < \langle\omega^2\rangle$, $\cdots\cdots$ $\omega^2 < 0.1\langle\omega^2\rangle$, $\cdot - \cdot - \cdot -$ $\omega^2 < 0.1\langle\omega^2\rangle$ and $\cos(\boldsymbol{\omega}, \boldsymbol{\lambda}_2) > 0.9$, Tsinober *et al.* 1995b, 1997. Just as in the previous figure this PDF is essentially different from the one for a Gaussian field even for very weak enstrophy ($\omega^2 < 0.1\langle\omega^2\rangle$).

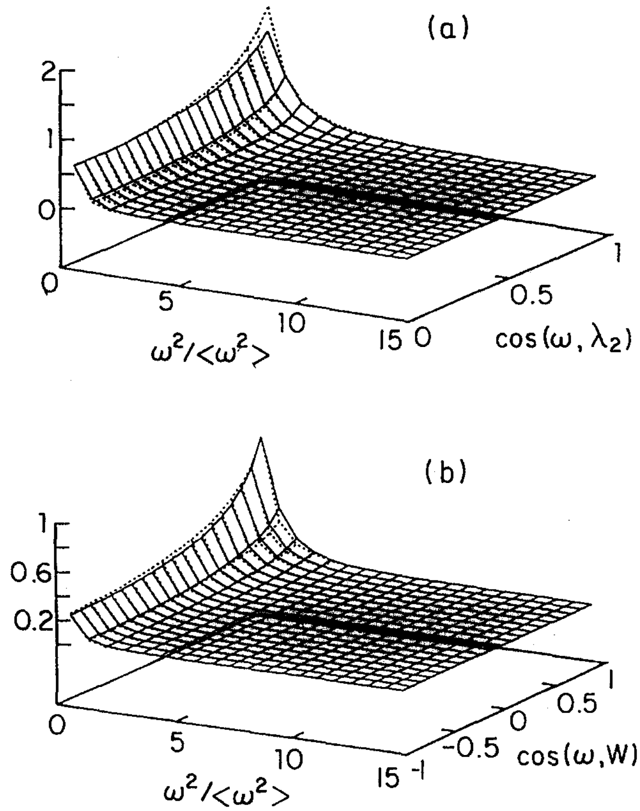

Fig. 5. – Joint PDFs of $\cos(\omega, \lambda_2)$ and ω^2 (a)), and $\cos(\omega, \mathbf{W})$ and ω^2 (b)) for grid turbulence (———) and DNS (- - - - -), $Re_\lambda \approx 75$, Tsinober *et al.* 1997. As mentioned in the text the *joint* PDF shown in (a) does not contradict the generally known result about the tendency of alignment between ω and λ_2 for large ω^2 : the ratio between the two values of the joint PDF at large and small $\cos(\omega, \lambda_2)$ is large for *large* ω^2. This is seen when the vertical axis in (a) is plotted as a logarithm as shown in Figure 6.

very similar results, e.g. such as shown above, were obtained in numerical simulations of the Euler equations (Bell and Marcus 1992, Brachet *et al.* 1992, Chorin 1994, Fernadez *et al.* 1995, Ohkitani 1993, 1997, Ohkitani and Kishiba 1995, Porter *et al.* 1998, Pumir and Siggia 1990) thereby showing that - at least qualitatively - they should be valid at $Re \gg 1$.

4. Strained vortical (Burgers-like) objects

The main feature and shortcoming of these objects (straight strained vortices) is that they possess *one-*dimensional vorticity and therefore zero curvature of vortex lines. Though the relation between vorticity and strain is essentially nonlocal '*the presence of a strained vortex itself modifies the local strain field*' (Moffatt *et al.* 1994, p.242) – after all both are composed of derivatives of the same velocity field. However, the special feature of the *straight* strained vortices is that they are impotent in the sense that they do not change that part of the strain by which they are strained themselves: this part of strain is prescibed *a priori, i.e.* it is independent and decoupled from their vorticity. These vortices only change that part of their strain which is not reacting back on their vorticity. In other words there is only a one way interaction: the vorticity is strained by that part of strain which does not 'know' anything about the vorticity. In this sense such vortices are passive. In other words, the essential ingredient of nonlinearity, the main feature of true 'genuine' nonlinear interaction — the self-amplification — is absent in these objects. In this sense the nonlinearity is reduced in these objects.

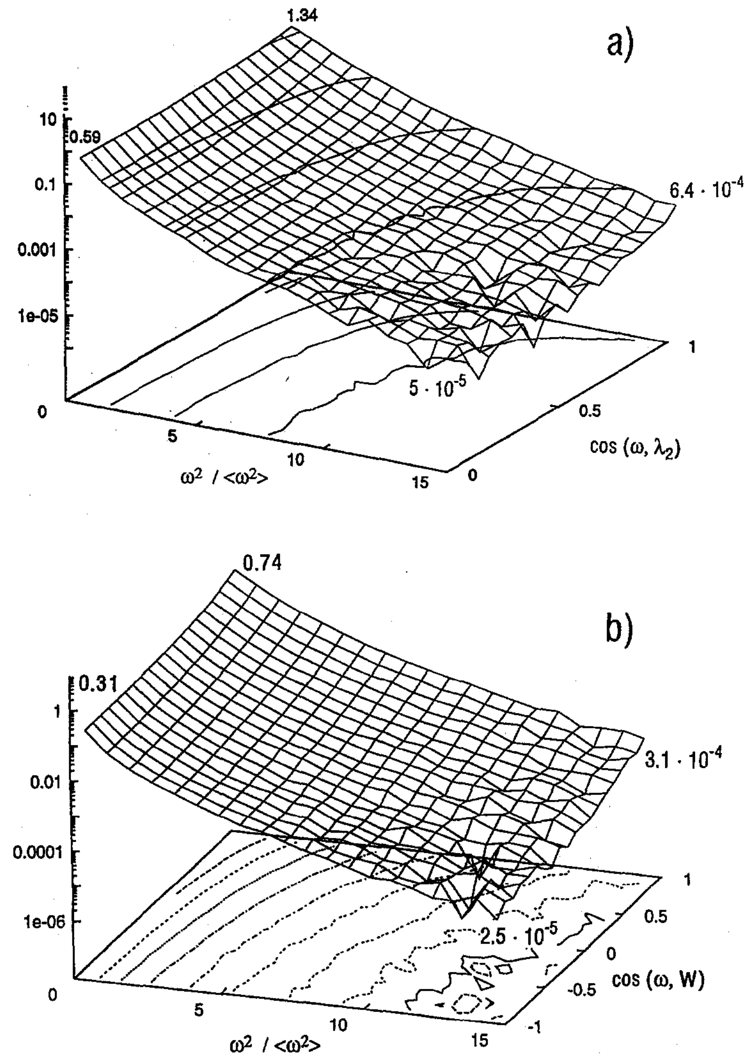

Fig. 6. – As for in Figure 5 for grid turbulence with the vertical axis plotted as a logarithm. $\mathrm{Re}_\lambda \approx 75$.

This property is directly related to zero curvature of vortex lines in straight strained vortices — the 'genuine' nonlinearity is present only in regions with nonvanishing curvature. This is what is observed when looking for (apparent) singularities of the Euler equations (Grauer and Sideris 1995, Kerr 1993, 1996, Novikov 1990, Pumir and Siggia 1990) and vortex reconnection (Fernandez *et al.* 1995, Kida and Takaoka 1994).

As noted in the introduction similar objects (*i.e.* regions with concentrated vorticity with small curvature) in real turbulent flows seem to be mostly the result, the consequence, rather than dominating factor of the turbulence dynamics. Possessing (almost) maximal enstrophy they are in an approximate equilibrium in the sense that their fairly large (but not largest!, *see* section 6) enstrophy generation is approximately balanced by the viscous reduction and in this sense they are less active than those regions possessing much larger (apparently maximal) enstrophy generation which is considerably larger than its viscous reduction. This is seen from the comparison of the rate of enstrophy generation $\alpha \equiv \omega_i \omega_k s_{ik}/\omega^2$ and its viscous reduction $\nu\omega_i \nabla^2 \omega_i/\omega^2$ in slots of ω and s as shown in Figure 7 (Tsinober *et al.* 1997, 1998). Indeed, the imbalance between stretching and viscous terms in slots of s is much *larger* than in slots of ω. This difference is especialy large at large values of of ω and s. This means that the time scale estimated from the imbalance of stretching and viscous

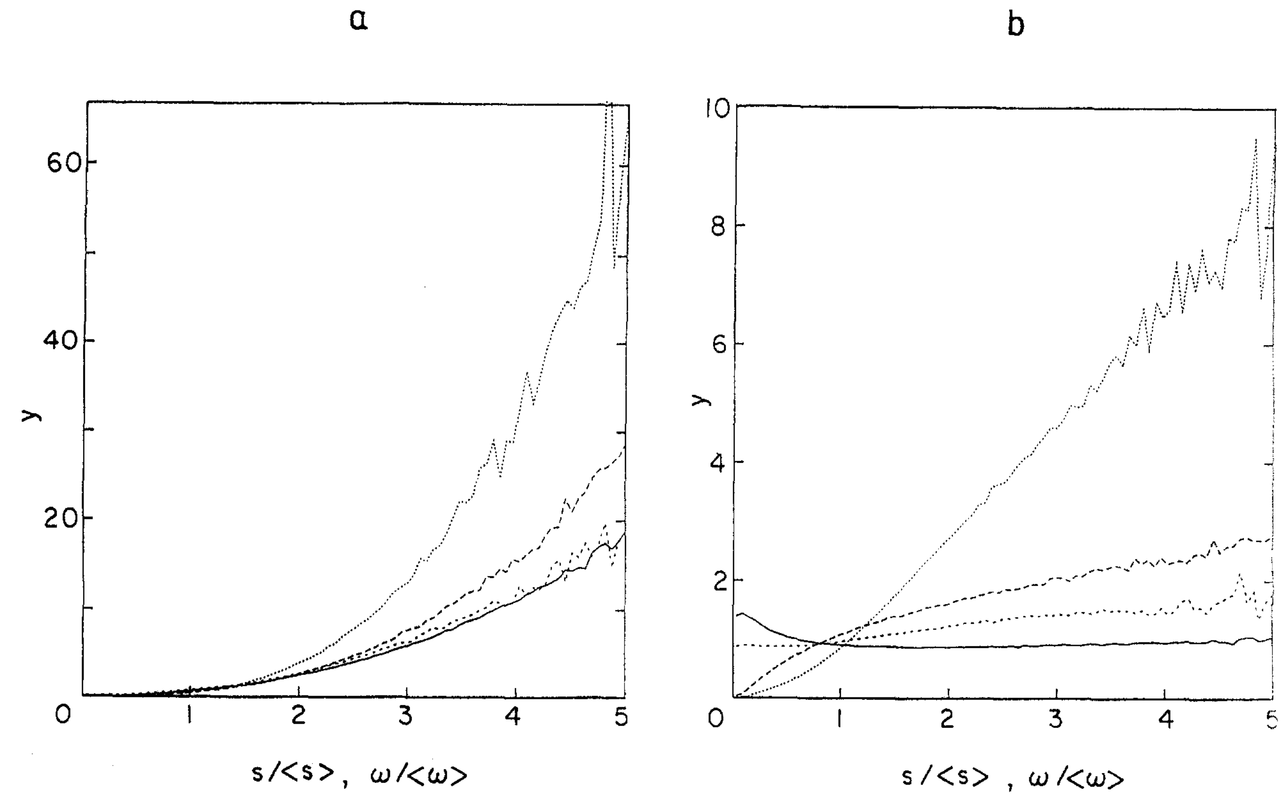

Fig. 7. – Comparison of the enstrophy generation $\sigma \equiv \omega_i\omega_k s_{ik}$ (a)) and its rate $\alpha \equiv \omega_i\omega_k s_{ik}/\omega^2$ (b)) with their viscous reduction $\nu\omega_i\boldsymbol{\nabla}^2\omega_i$ and $\nu\omega_i\boldsymbol{\nabla}^2\omega_i/\omega^2$ in *slots* of ω and s;
a): – – – – – $y = \langle\sigma|_\omega\rangle/\langle\sigma\rangle$, ———— - $y = \langle\omega_i\nabla^2\omega_i|_\omega\rangle/\langle\omega_i\nabla^2\omega_i\rangle$,
· · · · · – $y = \langle\sigma|_s\rangle/\langle\sigma\rangle$, - - - - - $y = \langle\omega_i\nabla^2\omega_i|_s\rangle/\langle\omega_i\nabla^2\omega_i\rangle$:
b): – – – – – $y = \langle\alpha|_\omega\rangle/\langle\alpha\rangle$, ———— - $y = \langle(\omega_i\nabla^2\omega_i/\omega^2)|_\omega\rangle/\langle\omega_i\nabla^2\omega_i\omega^2\rangle$,
· · · · · - $y = \langle\alpha|_s\rangle/\langle\alpha\rangle$, - - - - - $y = \langle(\omega_i\nabla^2\omega_i\omega^2)|_s\rangle/\langle\omega_i\nabla^2\omega_i\omega^2\rangle$.

terms $\omega^2\{D_t(\omega^2/2)\}^{-1} \approx \{\omega_i\omega_k s_{ik}/\omega^2 + \nu\omega_i\boldsymbol{\nabla}^2\omega_i/\omega^2\}^{-1}$ in slots of ω is much larger than such a time scale in slots of s, *i.e.* the life time of regions with concentrated vorticity is large comparing to that of the regions with large strain (rate of energy dissipation). This explains – at least in part – the observability of the regions with concentrated vorticity and the difficulties in observing the regions with large dissipation (but *see* Schwarz 1990). It is noteworthy that these objects in real turbulent flows possess essentially nonvanishing curvature (Galanti *et al.* 1996, Tsinober *et al.* 1998) so that the self-amplification of their vorticity does not vanish as for perfectly straight ones.

5. Reduction of nonlinearity

This notion has several aspects, all of them directly related to geometrical statistics. One of the simplest aspects concerns the magnitude of the vortex stretching and enstrophy generation terms, i.e. $W \equiv |\omega_j s_{ij}|$ and $\omega_i\omega_j s_{ij}$ in (1) and (2). Their magnitude is expected to be smaller than ω^2 and ω^3 respectively due to reduction of nonlinearity in long, thin tubes-filaments-worms which are believed to be in some sense locally quasi-one-dimensional (Frisch 1995), i.e. that nonlinearity is stronger *outside* of these structures. Hence the term depletion (expulsion) of nonlinearity. Following this line one would expect that in regions with strong alignment between vorticity $\boldsymbol{\omega}$ and the intermediate eigenvector $\boldsymbol{\lambda}_2$, vortex stretching and enstrophy generation should decrease

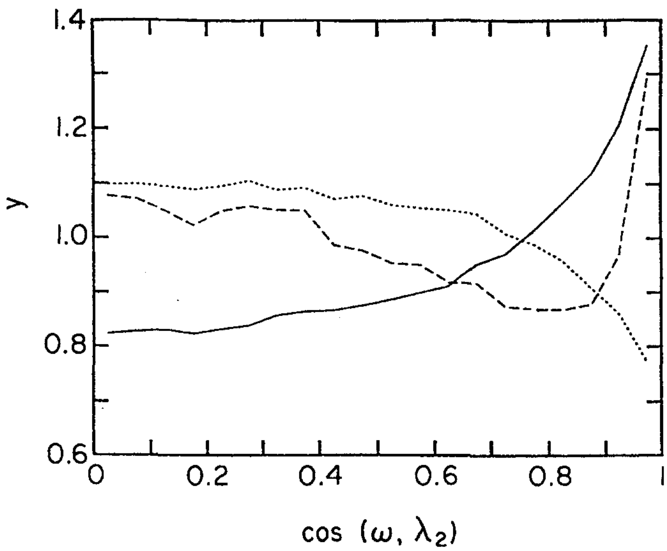

Fig. 8. – Mean values of enstrophy generation $\sigma \equiv \omega_i\omega_j s_{ij}$ (– – – –), magnitude of vortex stretching vector $W \equiv |\omega_j s_{ij}|$ (· · · · ·), and the intermediate eigenvalue Λ_2 (———) of the rate of strain tensor s_{ik} in *slots* of $\cos(\omega, \lambda_2)$, Tsinober *et al.* 1995b, 1997.

as $|cos(\boldsymbol{\omega}, \boldsymbol{\lambda}_2)|$ increases. Indeed W decreases somewhat but remains essentially finite. However, contrary to the above expectation the enstrophy generation *increases* in slots of $|\cos(\boldsymbol{\omega}, \boldsymbol{\lambda}_2)|$ and becomes *maximal* at $|\cos(\boldsymbol{\omega}, \boldsymbol{\lambda}_2)| \sim 1$ (*Fig.* 8). In other words in these regions the rate of creation of enstrophy ω^2 is at its largest and in this sense the nonlinearity is *stronger* and not weaker than in, at least, some of their background using $|\cos(\boldsymbol{\omega}, \boldsymbol{\lambda}_2)|$ as a criterion. The above tendencies are stronger in regions with strong vorticity and survive in the background, e.g. regions of weak enstrophy (Tsinober *et al.* 1997).

Note that none of the quantities $\omega_i\omega_j s_{ij}$, W, Λ_2 become small for $\cos(\boldsymbol{\omega}, \boldsymbol{\lambda}_{int}) \sim 1$ indicating that the flow does not become locally two-dimensional. In particular, it is important that in these regions the intermediate strain (*i.e.* Λ_2) is positive and is also increasing with $|cos(\boldsymbol{\omega}, \boldsymbol{\lambda}_2)|$, which corresponds to strong straining in these regions (cf. with pure two-dimensional flow in which $\Lambda_2 \equiv 0$). Thus one can speculate that there is a tendency to 'localization of nonlinearity' in space which, somewhat paradoxically, is sustained by nonlocal effects due nonlocal relation between strain and vorticity and due to pressure ('nonlocal localization'), *see* section 8. Note, that the claim of 'localization of nonlinearity' is supported by the behaviour of Λ_2/s in slots of $|cos(\boldsymbol{\omega}, \boldsymbol{\lambda}_2)|$, which is similar to that for λ_2 shown in Figure 9. In order to get more insight it is necessary to look into more subtle aspects of geometrical statistics than just single space/time point alignments. For the moment is it is clear that 'simple' structures in three-dimensional turbulence are qualitatively different from those in two-dimensional turbulence in which the nonlinearity is really depleted in such structures — however, in three-dimensional turbulence such structures do not seem to be good candidates to look for depletion of nonlinearity (Tsinober *et al.* 1995a,b 1997). Nevertheless, taking the enstrophy generation $\omega_i\omega_j s_{ij}$ as a measure of nonlinearity the objects with strong alignment between $\boldsymbol{\omega}$ and $\boldsymbol{\lambda}_2$ appear to be not the most nonlinear, since their enstrophy generation $\omega_i\omega_j s_{ij}$ comes mostly from the nonlocal effects and not from self-stretching (Jimenez *et al.* 1993, Tsinober *et al.* 1995a,b, 1997, Tsinober 18). The regions with strongest enstrophy generation are discussed in the next section.

Among other aspects of the problem of reduction of nonlinearity is the comparison of nonlinearities in real turbulent flows with their Gaussian counterparts (Kraichan and Panda 1988), which is meaningful for even moments only, e.g. $\langle|\mathbf{u} \times \boldsymbol{\omega} - \nabla(p + \frac{1}{2}u^2)|\rangle/\langle|\mathbf{u} \times \boldsymbol{\omega} - \nabla(p + \frac{1}{2}u^2)|\rangle_{Gaussian} < 1$ ($\sim 0.5 \div 0.6$) (Kraichnan and Panda 1988, Chen *et al.* 1989); $\langle W^2\rangle/\langle W^2\rangle_{Gaussian} < 1$ ($\sim 0.7 \div 0.8$) (Kerr 1985, Tsinober *et al.* 1992). In this sense nonlinearity is reduced. However, in the sense of odd moments the real nonlinearity is 'infinitely'

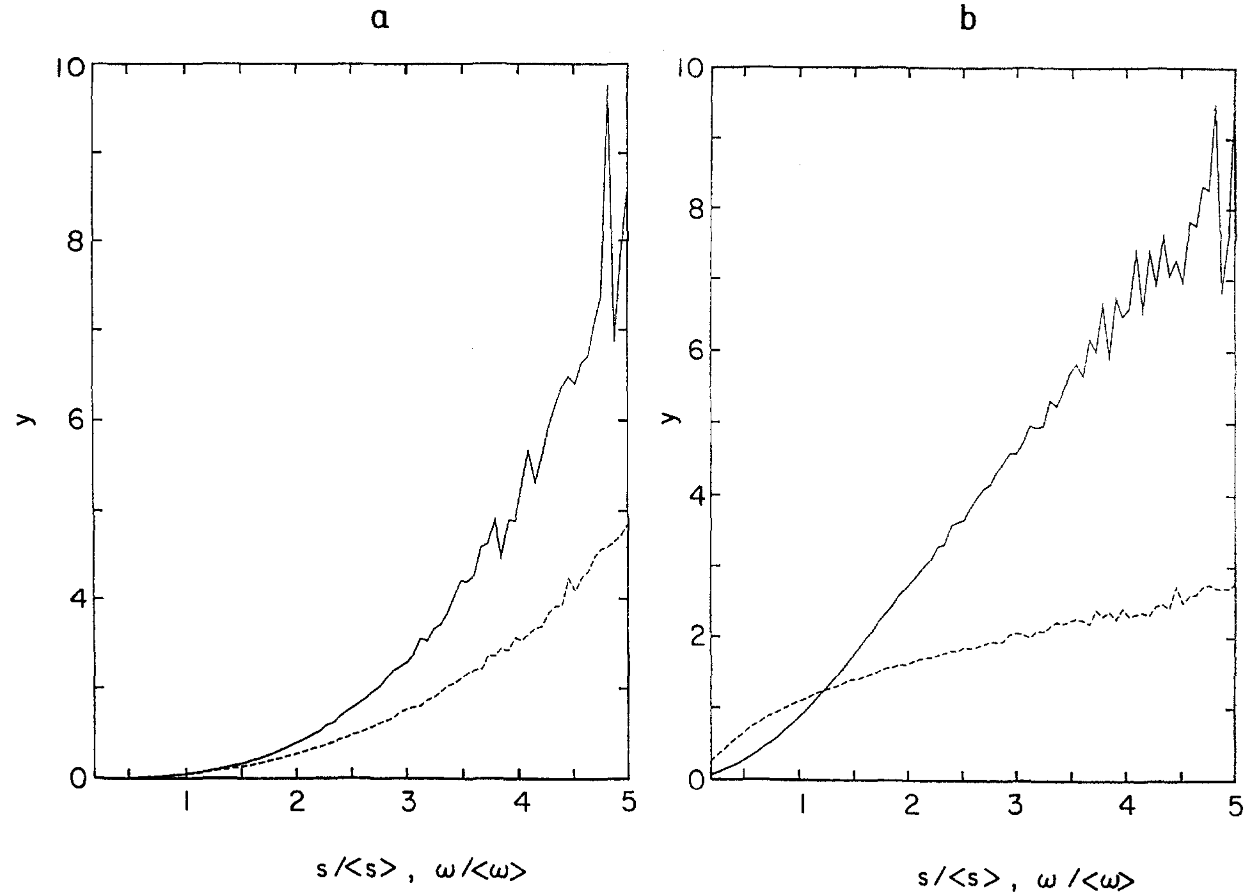

Fig. 9. – Comparison of mean values of enstrophy generation σ (*a*)) and its rate α (*b*)) in *slots* of ω and s, Tsinober *et al.* 1997.
(*a*)): - - - - - $y = \langle \sigma|_\omega \rangle / \langle \sigma \rangle$, ——— - $y = \langle \sigma|_s \rangle / \langle \sigma \rangle$;
(*b*)): - - - - - $y = \langle \alpha|_\omega \rangle / \langle \alpha \rangle$, ——— - $y = \langle \alpha|_s \rangle / \langle \alpha \rangle$.

larger, since for a Gaussian velocity field the odd moments vanish, e.g. velocity structure functions of odd order $S_{2n+1}(r) = \langle \{[\mathbf{u}(\mathbf{x}+\mathbf{r}) - \mathbf{u}(\mathbf{x})] \cdot \mathbf{r}/r\}^{2n+1} \rangle$, enstrophy generation $\langle \omega_i \omega_k s_{ik} \rangle$ and its rate $\langle \omega_i \omega_k s_{ik}/\omega^2 \rangle$ and many others. In other words build up of *odd* moments such as $\langle \omega_i \omega_k s_{ik} \rangle > 0$ is a manifestation of nonlinearity of turbulence along with being the manifestation of its structure. The nonzero $\langle \omega_i \omega_k s_{ik} \rangle$ is associated with the strict alignment between $\boldsymbol{\omega}$ and $\mathbf{W}$ and in this sense this alignment is enhancing the nonlinearity. As is seen from Figure 4 this alignment is significant throughout *all* the regions of turbulent flow. On the other hand, the alignment between $\mathbf{u}$ and $rot\ \boldsymbol{\omega}$ is reducing $\langle \omega_i \omega_k s_{ik} \rangle$. Indeed, since $\langle \omega_i \omega_k s_{ik} \rangle = \langle \boldsymbol{\omega} \cdot (\mathbf{u} \times \omega) \rangle = \langle rot \boldsymbol{\omega} \cdot (\mathbf{u} \times \boldsymbol{\omega}) \rangle = -\ \langle \boldsymbol{\omega} \cdot (\mathbf{u} \times rot \boldsymbol{\omega}) \rangle$, and $-\ \langle \mathbf{u} \cdot rot\ \boldsymbol{\omega} \rangle \equiv \langle \omega \rangle^2 > 0$ there is a tendency for (anti-)alignment between $\mathbf{u}$ and $rot\ \boldsymbol{\omega}$ reducing the magnitude of $\mathbf{u} \times rot\ \boldsymbol{\omega}$ and thereby of $\langle \omega_i \omega_k s_{ik} \rangle$. Though this is a purely kinematic effect it is directly related to the dissipative nature of turbulent flows, since the mean dissipation $\langle \epsilon \rangle \simeq \nu \langle \omega \rangle^2$.

On other aspects of reduction of nonlinearity *see* Moffatt 1990, Shtilman 1992, Shtilman and Polifke 1989, Tsinober 1990, Tsinober *et al.* 1995a,b and references therein.

6. Regions of strongest vorticity/strain interaction

The next important point is that at least in quasi-isotropic flows the largest contribution to the enstrophy generation $\omega_i\omega_j s_{ij} = \omega_i^2\Lambda_i\cos^2(\boldsymbol{\omega},\boldsymbol{\lambda}_i)$ comes from the regions associated with the *largest* eigenvalue Λ_1 of the rate of strain tensor s_{ij} (Tsinober *et al.* 1992, 1995b, 1997b, 1998, Vincent and Meneguzzi 1994) and not from those associated with the *intermediate* eigenvalue Λ_2 to which belong the regions of concentrated vorticity. Namely the ratio of $\langle\omega_1^2\Lambda_1\cos^2(\boldsymbol{\omega},\boldsymbol{\lambda}_1)\rangle$ to $\langle\omega_2^2\Lambda_2\cos^2(\boldsymbol{\omega},\boldsymbol{\lambda}_2)\rangle$ is roughly $2:1$. The same is true of $\alpha = \Lambda_i\cos^2(\boldsymbol{\omega},\boldsymbol{\lambda}_i)$ (*see* Table II).

TABLE II. – Contribution to the total mean enstrophy generation $\langle\sigma\rangle \equiv \langle\omega_i^2\Lambda_i\cos^2(\boldsymbol{\omega},\boldsymbol{\lambda}_i)\rangle$ and its rate $\langle\alpha\rangle \equiv \langle\lambda_i\cos^2(\boldsymbol{\omega},\boldsymbol{\lambda}_i)\rangle$ from the terms corresponding to the eigenvalues Λ_i of the rate of strain tensor s_{ij}.

	$\langle\omega^2\Lambda_1\cos^2(\boldsymbol{\omega},\boldsymbol{\lambda}_1)\rangle$	$\langle\omega_2^2\Lambda_2\cos^2(\boldsymbol{\omega},\boldsymbol{\lambda}_2)\rangle$	$\langle\omega_3^2\Lambda_3\cos^2(\boldsymbol{\omega},\boldsymbol{\lambda}_3)\rangle$
DNS	1.06	0.51	-0.51
Grid	1.17	0.39	-0.56
	$\langle\Lambda_1\cos^2(\boldsymbol{\omega},\boldsymbol{\lambda}_1)\rangle$	$\langle\Lambda_2\cos^2(\boldsymbol{\omega},\boldsymbol{\lambda}_2)\rangle$	$\langle\Lambda_3\cos^2(\boldsymbol{\omega},\boldsymbol{\lambda}_3)\rangle$
DNS	1.47	0.49	-0.97
Grid	1.17	0.46	-0.63

This shows that there exist regions (intense and weak — both structured and dynamicaly active) other than concentrated vorticity regions, which at least in the above sense are dynamicaly more important (Tsinober *et al.* 1995, 1997a, b). These regions are associated mainly with largest strain rather than enstrophy (Ruetsch and Maxey 1991, Tsinober *et* 1997a), strong tendency of alignment between $\boldsymbol{\omega}$ and $\boldsymbol{\lambda}_1$ (Tsinober *et al.* 1995, 1997b), and fairly large curvature of vorticity lines (Tsinober *et al.* 1998). These regions are characterised by the largest, apparently maximal, enstrophy generation and its rate (*see Fig.* 9, the behaviour of $|\mathbf{W}|$ in slots of ω and s is essentially the same) which are much larger than their viscous reduction as discussed in section 4 and shown in Figure 7. As implied by the results shown in table 1 these regions are associated with a strong tendency for alignment between $\boldsymbol{\omega}$ and the largest eigenvalue Λ_1 of the rate of strain tensor s_{ij} as illustrated in Figure 10 (Tsinober *et al.* 1995b, 1997). Similarly the dependence of enstrophy generation $\sigma \equiv \omega_i\omega_j s_{ij}$ and its rate $\alpha \equiv \Lambda_i\cos^2(\boldsymbol{\omega},\boldsymbol{\lambda}_i)$ on ω and on $s \equiv (s_{ij}s_{ij})^{1/2}$ is qualitatively different for small and large curvature of vortex lines in such a way that the nonlinearity is manifested stronger in regions of large curvature (Tsinober *et al.* 1998). In particular, the disparity in the behaviour of σ and α in slots of ω and s becomes larger at small curvature, whereas at large curvature the dependence of σ and α on ω and s is very similar. This last fact is a reflection of the stronger interaction of vorticity and strain in regions with large curvature and positive α.

The regions just discussed comprise a *subset* of larger regions dominated by strain. Namely, these are the regions with large vortex line curvature. There exist at least two other kinds of strain dominated regions: those with small curvature of vortex lines, which wrap around the vorticity dominated regions (tubes/worms) (Ruetsch and Maxey 1991), which contribute also mostly to the alignment of $\boldsymbol{\omega}$ and $\boldsymbol{\lambda}_2$, and regions with large magnitude of Λ_3 and large negative α, in which significant vortex *compressing, tilting and folding* occur.

7. Curvature and folding

One of the most basic phenomena in turbulence is the predominant vortex stretching, which is manifested in predominant enstrophy generation $\sigma \equiv \omega_i\omega_j s_{ij}$ (so that $\langle\sigma\rangle > 0$). This process consists not only of vortex stretching ($\sigma > 0$), but also of vortex compressing ($\sigma < 0$), and cannot occur (in a finite volume) without its concomitant - the process of vortex folding (Chorin 1982, also 1994 and references therein; the term folding was

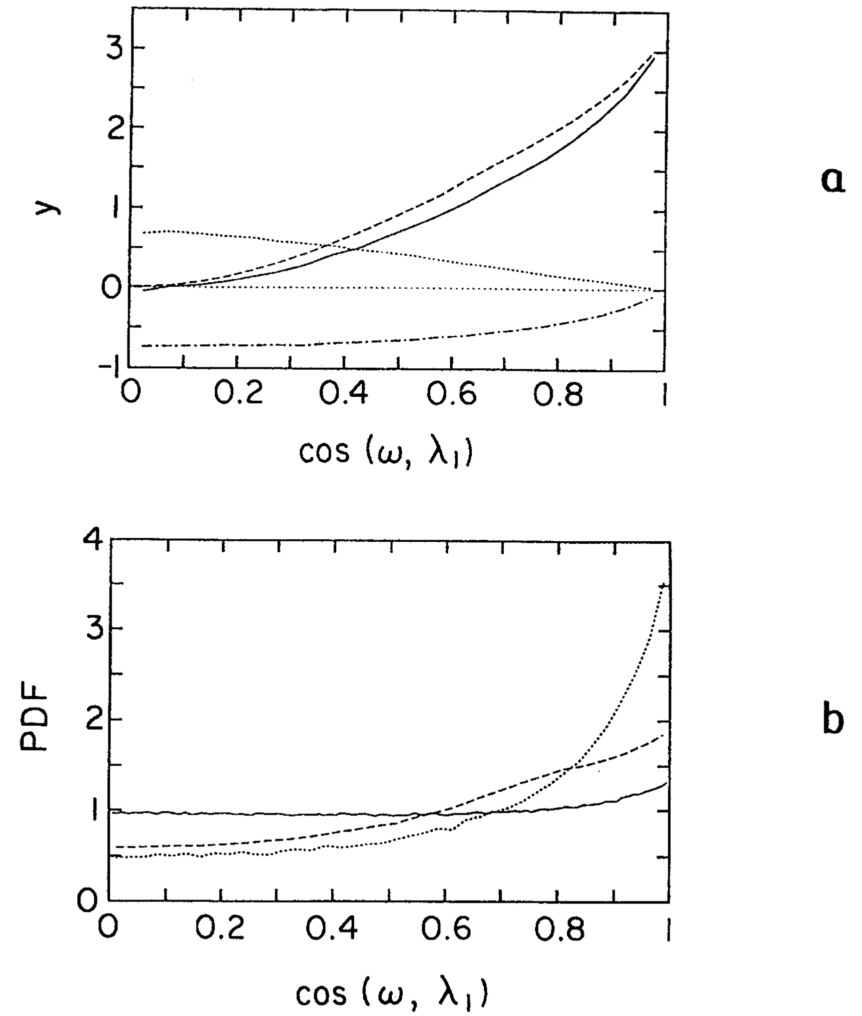

Fig. 10. – *a*) Mean values of enstrophy generation $\sigma \equiv \omega^2 \lambda_i \cos^2(\boldsymbol{\omega}, \boldsymbol{\lambda}_i)$ and $\sigma_\alpha = \lambda_\alpha \cos^2(\boldsymbol{\omega}, \boldsymbol{\lambda}_\alpha)$, $\alpha = 1, 2, 3$ (no summation over α) in *slots* of $\cos(\boldsymbol{\omega}, \boldsymbol{\lambda}_1)$: ——— - $y = \langle \sigma|_{\cos(\boldsymbol{\omega}, \boldsymbol{\lambda}_1)} \rangle / \langle \sigma \rangle$, – – – – – $y = \langle \sigma_1|_{\cos(\boldsymbol{\omega}, \boldsymbol{\lambda}_1)} \rangle / \langle \sigma \rangle$, · · · · · - $y = \langle \sigma_2|_{\cos(\boldsymbol{\omega}, \boldsymbol{\lambda}_1)} \rangle / \langle \sigma \rangle$, – · – · – · – $y = \langle \sigma_3|_{\cos(\boldsymbol{\omega}, \boldsymbol{\lambda}_1)} \rangle / \langle \sigma \rangle$;
b) PDFs of $\cos(\boldsymbol{\omega}, \boldsymbol{\lambda}_1)$ for large σ : ——— - whole field, – – – – - $\sigma > 0$, $P = 0.70$ (70% of points) , · · · · · $\sigma > 5\langle \sigma \rangle$, $P = 0.06$ (6% of points). Grid turbulence $\mathrm{Re}_\lambda \approx 75$, Tsinober *et al.* 1997.

introduced by Reynolds (1894) in the context of folding of material lines). Hence, the importance of looking at properties of turbulent flow in regions with large curvature and $\sigma < 0$, which typically occupy 1/3 of the whole flow volume, and for the evidence and characterisation of the vortex folding in three-dimensional turbulence. It is reasonable to associate this last process with large curvature of vortex lines and similar quantities, which should reflect their folding - at least the resulting aspect of this process. Then the second main question is about the properties of curvature and the relation between curvature and dynamically relevant quantities such as enstrophy ω^2, enstrophy generation σ, rate of enstrophy generation $\alpha \equiv \sigma/\omega^2$ and relations such as various alignments. Of course, the ultimate clarification of such relations can be obtained by looking at global properties. One can hope that some insights can be gained from local analysis, i.e. from working with point quantities at a particular time moment. We offer a few typical results relevant to the theme of this paper. More details are given in Tsinober *et. al* 1998.

In a simplified form the logic is that strong stretching results in strong vorticity: indeed regions with strong vorticity are known to be tube-like with small curvature as observed in a number of numerical simulations cited above (*see* also Galanti *et. al* 1996). However, a closer inspection shows that matters are much more complicated (*Fig.* 11) due to a number of qualitative differences between material and vortex lines (*see* Appendix). One can see that indeed, the curvature decreases in slots of ω^2. However, this behaviour is practically the same for the whole field, for positive and for *negative* rate of enstrophy generation α (the reader is reminded again that typically a turbulent flow field consists of about 2/3 points with $\alpha > 0$ and 1/3 - with $\alpha < 0$). This last fact, *i.e.* the behaviour of curvature C versus ω^2 for *negative* rate of stretching $\alpha < 0$ and strong increase of curvature with strain (*Fig.* 11) undermines the simple analogy with the behaviour of material lines in turbulent flows (*see* Appendix). Similarly, as is expected the curvature of vortex lines is *increasing* with $|\alpha|$ for $\alpha < 0$ due to the folding of vortex lines, but again, most interestingly the same behaviour of C is observed for $\alpha > 0$ due to self-induction (*Fig.* 12) unlike the case of material lines. This is consistent with the results of the comparison of dependence of enstrophy generation $\omega_i \omega_k s_{ik}$ and its viscous reduction $\nu \omega_i \nabla^2 \omega_i$ on ω^2 and s^2 and curvature C (*Fig.* 7).

The preferential alignment between $\boldsymbol{\omega}$ and $\boldsymbol{\lambda}_2$ is correlated with small curvature and there is no preferential alignment between $\boldsymbol{\omega}$ and $\boldsymbol{\lambda}_2$ at large curvature (*Fig.* 13).

The above shows that the 'most nonlinear' are the regions with large curvature, dissipation and preferable alignment between $\boldsymbol{\omega}$ and $\boldsymbol{\lambda}_1$, and not the regions of concentrated vorticity with small curvature and preferable alignment between $\boldsymbol{\omega}$ and $\boldsymbol{\lambda}_2$, such as the filaments observed in direct numerical simluations of Navier-Stokes equations (e.g. She 1991, She *et al.* 1991, She and Leveque 1994).

It is noteworthy that regions of concentrated vorticity are not free of vortex compression in the same proportion as the whole turbulent field (Jimenez and Wray 1994a,b), which is possibly associated with Kelvin waves along the worms (Verzicco *et al* 1995), and is consistent with the results shown in Figure 11.

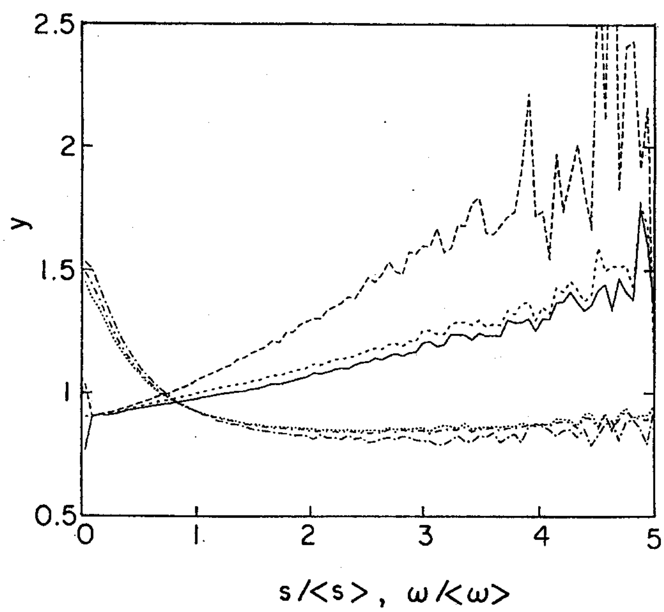

Fig. 11. – Mean values of curvature C of vortex lines in *slots* of ω and s for the whole field and for positive and negative rate of enstrophy generation α: – · – · – · – · · $y = \langle c|_\omega \rangle / \langle c \rangle$, · · · · · · $y = \langle c|_\omega \rangle / \langle c \rangle$ and $\alpha > 0$,— · — · — · — · · $y = \langle c|_\omega \rangle / \langle c \rangle$ and $\alpha < 0$;- - - - - $y = \langle c|_s \rangle / \langle c \rangle$, ———— - $y = \langle c|_s \rangle / \langle c \rangle$ and $\alpha > 0$, — — — — - $y = \langle c|_\omega \rangle / \langle c \rangle$ and $\alpha < 0$. DNS turbulence, $\mathrm{Re}_\lambda \approx 75$, Tsinober *et al.* 1998. Note the *qualitatively* different behaviour of curvature in *slots* of ω (decreasing) and in *slots* of s (increasing)

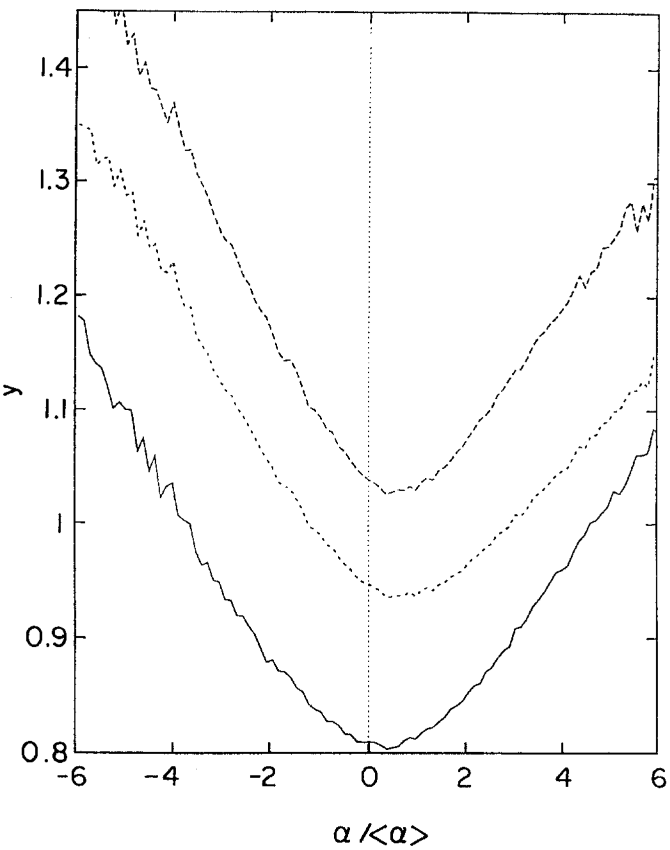

Fig. 12. – Mean values of curvature C of vortex lines in *slots* of rate of enstrophy generation α for DNS turbulence: - - - - - whole field, ———— - $\omega^2 > 2.5\langle\omega^2\rangle$, – – – – - $\omega^2 < 0.5\langle\omega^2\rangle$. $\mathrm{Re}_\lambda \approx 75$, Tsinober *et al.* 1998. Note that the curvature of vortex lines *increases* with $|\alpha|$ *both* for negative and positive α.

The issue of curvature of vortex lines and vortex folding is closely realted to that of vortex tilting according to the equation (4). For more on this aspect *see* Tsinober 1998, Tsinober *et al.* 1998.

8. Nonlocality

As mentioned in section 5 the localization of vorticity in vortex filaments is mostly sustained by the nonlocal effects in physical space, which keep all the regions in turbulent flow in continuous interaction and mutual transformation.

The well known property of nonlocality of NSE in physical space is two-fold. On one hand, it is due to pressure ('dynamic' nonlocality), since $\rho^{-1}\nabla^2 p = \omega^2 - 2s_{ij}s_{ij}$, so that pressure is nonlocal due to nonlocality of the operator ∇^{-2}. The nonlocality is strongly associated with essentially non-Lagrangian nature of pressure. For example, replacing in the Euler equations the pressure Hessian $\frac{\partial^2 p}{\partial x_i \partial x_j}$, which is both nonlocal and non-Lagrangian, by a local quantity $\frac{1}{3}\delta_{ij}\nabla^2 p = \frac{\rho}{6}\{\omega^2 - 2s_{ij}s_{ij}\}$ turns the problem into a local one and allows integration of the equations for the invariants of the tensor of velocity derivatives $\partial u_i/\partial x_j$ in terms of a Lagrangian system of coordinates moving with a particle (Blackburn *et al.* 1996, Cantwell 1992 and references therein). This means that nonlocality due to presure is essential for sustaining turbulence.

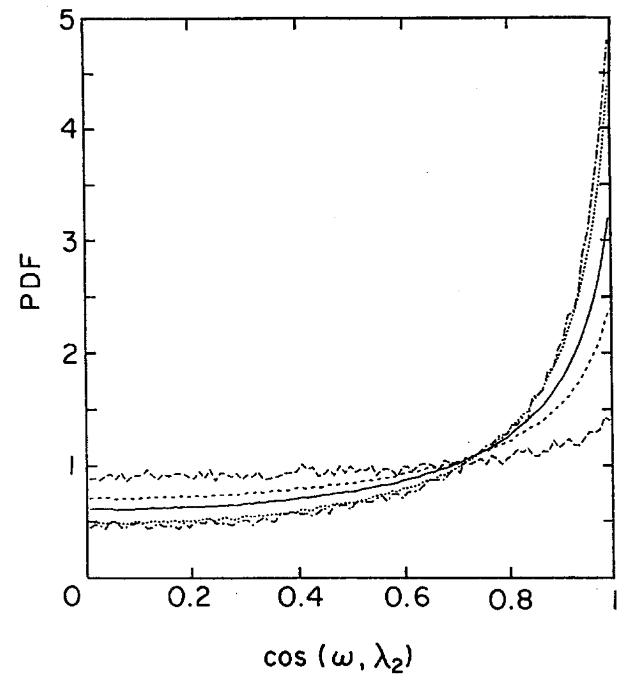

Fig. 13. – PDFs of the cosine of the angle between vorticity ω and the intermediate eigenvector λ_2 of the rate of strain tensor for DNS turbulence: ——— - whole field, — — — — - $C > 2.5\langle C\rangle$, - - - - - $C > \langle C\rangle$, · · · · · - $C < 0.5\langle C\rangle$, — · — · — · — · - $C < 0.2\langle C\rangle$; For $C > k\langle C\rangle$ $(k > 2.5)$ the behaviour is the same as for $k = 2.5$; $\mathrm{Re}_\lambda \approx 75$, Tsinober *et al.* 1998.

Similarly the equations for vorticity (1) and enstrophy (2) are nonlocal in ω since they contain the rate of strain tensor $s_{ij} = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}$, $\mathbf{u} = rot^{-1}\boldsymbol{\omega}$ due to nonlocality of the operator rot^{-1} ('kinematic' nonlocality). Both aspects are reflected in the equation for the rate of strain tensor s_{ij} (Yanitskii 1982)

$$D_t s_{ij} = -s_{ik}s_{kj} - \frac{1}{4}(\omega_i\omega_j - \omega^2\delta_{ij}) - \frac{\partial^2 p}{\partial x_i \partial x_j} + vt,$$

and for the third order quantities, e.g. $\omega_i\omega_j s_{ij}$, α, (Ohkitani and Kishiba 1995, Tsinober 1995, Tsinober *et al.* 1995b)

$$D_t\omega_i\omega_j s_{ij} = \omega_j s_{ij}\omega_k s_{ik} - \omega_i\omega_j \frac{\partial^2 p}{\partial x_i \partial x_j} + vt, \tag{5}$$

$$D_t\alpha = -2\alpha^2 + \Lambda_i^2\cos^2(\boldsymbol{\omega}, \boldsymbol{\lambda}_i) - \pi_i\cos^2(\boldsymbol{\omega}, \boldsymbol{\pi}_i) + vt, \tag{6}$$

where vt stands for viscous terms and $\pi_i, \boldsymbol{\pi}_i$ are respectively the eigenvalues and the eigenvectors of the pressure hessian $\frac{\partial^2 p}{\partial x_i \partial x_j}$.

It is seen from the equations (5), (6) that the rates of change of enstrophy generation $D_t\sigma$ and $D_t\alpha$ depend on the geometrical relations between vorticity $\boldsymbol{\omega}$ and both the eigenframe of the rate of strain tensor $\boldsymbol{\lambda}_i$ and that of the pressure hessian $\boldsymbol{\pi}_i$.

An important aspect is that equation (5) and a similar one for $s_{ik}s_{kj}s_{ij}$ contain two invariant quantities $\mathcal{I}_5 = \omega_i\omega_j \frac{\partial^2 p}{\partial x_i \partial x_j}$; $\mathcal{I}_6 = s_{ik}s_{kj} \frac{\partial^2 p}{\partial x_i \partial x_j}$ reflecting the nonlocal dynamical effects due to pressure and can be interpreted as interaction between vorticity and strain, and pressure. In particular equation (5) for the enstrophy generation $\sigma \equiv \omega_i\omega_j s_{ij}$ shows both aspects of nonlocality of vortex stretching process. The first term in (5) is strictly positive, $\omega_j s_{ij}\omega_k s_{ki} \equiv W^2 > 0$. This means that the nonlinear processes involving vortex stretching (or direct interaction of vorticity and strain) always tend to increase even the instantaneous enstrophy generation. Here also the term $W_3^2 = \omega_3^2\Lambda_3^2\cos^2(\boldsymbol{\omega}, \boldsymbol{\lambda}_3)$ associated with the negative eigenvector of the rate of strain tensor λ_3, *i.e.* vortex compressing or negative enstrophy production $\omega^2\lambda_3\cos(\boldsymbol{\omega}, \boldsymbol{\lambda}_3)$, makes a positive (!) contribution to the rate of change of enstrophy generation ($W_i^2 = \omega^2\Lambda_i^2\cos^2(\boldsymbol{\omega}, \boldsymbol{\lambda}_i)$). It is natural to call the term W^2 the inviscid rate of *increase* of the enstrophy generation term. However, the inviscid rate of *change* of enstrophy generation contains also a second term reflecting the interaction between vorticity and the pressure hessian $\frac{\partial^2 p}{\partial x_i \partial x_j}$. It appears that $\langle \omega_i\omega_j \frac{\partial^2 p}{\partial x_i \partial x_j}\rangle$ is positive and is about $\langle W^2\rangle/3$, *i.e.* in the mean the nonlinearity in (5) is reduced by this nonlocal term, since for a Gaussian velocity field $\langle \omega_i\omega_j \frac{\partial^2 p}{\partial x_i \partial x_j}\rangle \equiv 0$. Consequently the PDF of $\cos(\boldsymbol{\omega}, \mathbf{W}^{\Pi})$, $W_i^{\Pi} = \omega_j \frac{\partial^2 p}{\partial x_i \partial x_j}$, is strongly skewed just as the PDF of $\cos(\boldsymbol{\omega}, \mathbf{W})$ (Tsinober *et al.* 1995b, Tsinober and Shtilman 1998).

9. How much non-Gaussian is turbulence?

As mentioned above one of the most spectacular manifestations of the non-Gaussian nature of turbulence is the build up of *odd* moments of various quantities (*see* sections 2,3). It is for this reason the quantity $cos(\boldsymbol{\omega}, \mathbf{W}) = \omega_i\omega_j S_{ij}|\omega|^{-1}|W|^{-1}$ appeared so useful in the diagnostics of the non-Gaussian nature of the 'random' 'structureless' sea in turbulent flows.

A simple demonstration how the dynamics of turbulence makes it non-Gausssian can be seen from taking $\langle \ldots \rangle$ from the equation (5) (dropping the viscous term)

$$\frac{D}{Dt}\langle \omega_i\omega_j s_{ij}\rangle = \langle \omega_j s_{ij}\omega_k s_{ik}\rangle - \langle \omega_i\omega_j \frac{\partial^2 p}{\partial x_i \partial x_j}\rangle \tag{10}$$

For a Gaussian velocity field $\langle \omega_i\omega_j s_{ij}\rangle = 0$, $\langle \omega_i\omega_j \frac{\partial^2 p}{\partial x_i \partial x_j}\rangle = 0$, $\langle \omega_j s_{ij}\omega_k s_{ik}\rangle \equiv \langle W^2\rangle = \frac{1}{6}\langle \omega^2\rangle^2$ (Shtilman *et al.* 1993, Tsinober *et al.* 1995b). If the initial conditions are Gausssian the flow ceases to be Gausssian with *finite rate*. It is seen also from (10) that initially Gaussian and *potential* velocity field with small seeding of vorticity will produce - at least for a short time - an essentially positive enstrophy generation.

Turbulence – being essentially non-Gaussian – is such a rich phenomenon that it can 'afford' a number of manifestations (sometimes nontrivial), which are Gaussian-like. A recent example of such behaviour is shown in Figure 14. This example is interesting in that the Gaussian-like behaviour is exhibited by third order quantitites. For other examples see (Shtilman *et al.* 1993).

Many second order quantities in turbulent flows exhibit exponential tails in their PDF's. However, precisely the same behaviour is characteristic of a purely Gaussian velocity field. For example, PDF's of ω^2 and s^2 of a Gaussian velocity field have exponetial tails and their shape is very similar to that in real turbulent flows (Shtilman *et al.* 1993). Using the explicit expressions for the PDF's of ω^2 and s^2 from Shtilman *et al.* 1993 it is straightforward to obtain the values of flatness for ω^2 and s^2 for an arbitrary Gaussian velocity field. Alternatively the same result is obtained directly without invoking the explicit expressions for the PDF's of ω^2 and s^2, but using instead the decomposition rule for fourth-order moments. Namely, $F_{\omega^2} = \langle \omega^4\rangle/\langle \omega^2\rangle^2 = 5/3$ and $F_{s^2} = \langle s^4\rangle/\langle s^2\rangle^2 = 7/5$, *i.e.* the flattness of enstrophy is *greater* than that of total strain $\langle\omega^4\rangle/\langle\omega^2\rangle^2 - \langle s^4\rangle/\langle s^2\rangle^2 = 4/15$ [3]. Does one have to conclude from the above result that the enstrophy field is more intermittent than that of total strain in a *Gaussian* velocity field? Definitely not, since by definition Gaussian velocity fields lack any intermittency. This example shows that even moments *only* are not sufficiient for characterisation of the non-Gaussian nature and intermittency of turbulence. The same is true of pressure, which in addition has a strongly negatively skewed PDF, though in this case it is not easy to demonstrate this directly due to the nonlocal nature of the ∇^{-2} operator (Holzer and Siggia 1994). It is much easier to do this by looking at $\nabla^2 p$. Using the same method as in (Shtilman *et al.* 1993) the PDF $\mathcal{P}(x)$, $x = \frac{\nabla^2 p}{\rho\langle\omega^2\rangle} = \frac{\omega^2 - 2s_{ij}s_{ij}}{2\langle\omega^2\rangle}$, is expressed in the following way (Spector 1996, private communication)

$$\mathcal{P}(x) = \{3^{1/2}5^{5/2}\}/(4\pi)\; x^2\; e^{x}\; [K_2(4x) - K_1(4x)], \qquad x < 0,$$

which for large $|x|$ has the asymptotics $\sim |x|^{1/2}\; e^{-3|x|}$.

And

$$\mathcal{P}(x) = \{3^{1/2}5^{5/2}\}/(4\pi)\; x^2\; e^{-|x|}[K_2(4x) + K_1(4x)], \qquad x > 0,$$

which for large x has the asymptotics $\sim x^{3/2}\; e^{-5x}$.

So, as expected, the distribution of $\nabla^2 p$ is asymmetric even for a Gaussian velocity field. This result is in good quantitative agreement with values from laboratory and DNS experiments (*Fig.* 15), showing that these effects are mostly of kinematical nature as many others (Shtilman *et al.* 1993).

In this sense the non-Gaussian strongly intermittent behaviour of passive objects (scalars, vectors) in Gaussian or any other *a priory* prescribed random velocity field is a kinematic effect.

10. Concluding remarks

As pointed out at the outset of this paper the answer to the question posed in its title is that – though important – the regions of concentrated vorticity are not as important as is commonly believed, and that regions *other* than concentrated vorticity play essential role in the dynamics of turbulent flows. The main points supporting this claim are summarised in this section. In particular

$\mathcal{A}$. Structure in quasi-homogeneous/isotropic turbulent flow is associated with strong alignments rather than with, say, strong vorticity only. Consequently, practically *all* regions in turbulent flow rather than just those with intense vorticity are spatially structured. In, fact the whole flow field - even with weakest measurable vorticity - is not structureless, strongly non-Gaussian and dynamically not passive. This is true of almost all the regions with 'weak' excitation in some sense.

$\mathcal{B}$. The *locally* quasi-two-dimensional regions corresponding to large $\cos(\boldsymbol{\omega}, \boldsymbol{\lambda}_2)$ are qualitatively different from purely two-dimensional ones in that they posses essentially nonvanishing enstrophy generation and an intermediate eigenvalue of the rate of strain tensor. Moreover, in these regions both are *larger* than in the whole field. Hence 'simple' structures in three-dimensional turbulence are qualitatively different from those in two-dimensional turbulence in which the nonlinearity is really depleted in such structures — in three-dimensional turbulence such structures do not seem to be good candidates for depletion of nonlinearity. Nevertheless, taking the enstrophy generation $\omega_i\omega_j s_{ij}$ (and/or its rate) as a measure of nonlinearity, the objects with strong alignment between $\boldsymbol{\omega}$ and $\boldsymbol{\lambda}_2$ appear to be not the most nonlinear, since their enstrophy generation $\omega_i\omega_j s_{ij}$ comes mostly from the nonlocal effects and not from self-stretching/self-amplification.

[3] This is precisely the extreme of the 'generic' inequality obtained in Chen and Chen 1998 in somewhat different way using the the decomposition rule for fourth-order moments.

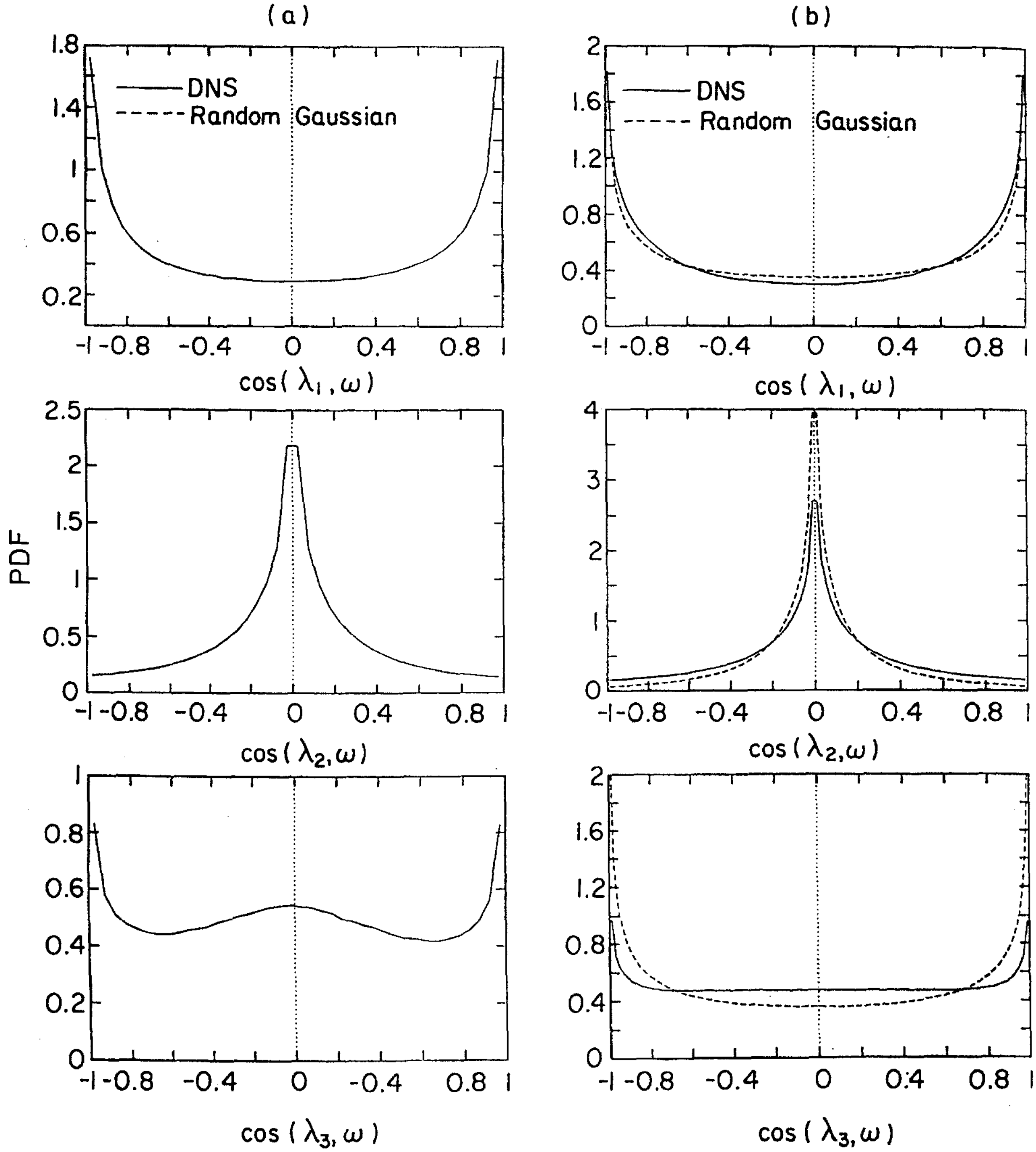

Fig. 14. – PDFs of the cosine of the angle between vortex stretching vector $\mathbf{W}, W_i = \omega_i s_{ij}$, and the eigenvectors λ_i of the rate of strain tensor; *a*) - grid turbulence, *b*) - DNS and random Gaussian (– – – –). $\mathrm{Re}_\lambda \approx 75$. The behaviour shown for $\cos(\omega, \lambda_2)$ seems to contradict that shown in Figures 3 and 4, since there exists a tendency for alignment between ω and $\mathbf{W}$ and between ω and λ_2. However, a closer inspection shows that the alignments shown in this figure and in Figures 3 and 4 are associated mostly with *different* regions in the flow.

C. The largest enstrophy generation occurs in the regions of strongest (local) vorticity/strain interaction, which are different from those with concentrated vorticity. These regions contribute most to the enstrophy generation and at least in this respect are dynamicaly more important than those of concentrated vorticity. These regions

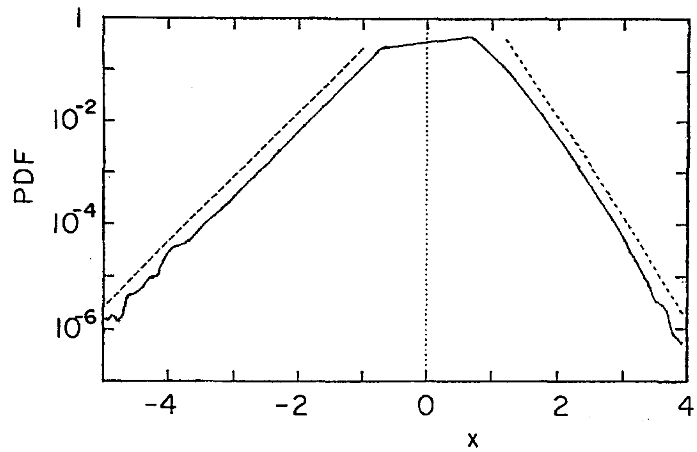

Fig. 15. – PDF of the Laplacian of pressure $x = \frac{\nabla^2 p}{\rho\langle\omega^2\rangle} = \frac{\omega^2 - 2s_{ij}s_{ij}}{2\langle\omega^2\rangle}$.
——— - grid turbulence, $Re_\lambda \approx 75$; - - - - - - slope + 4.5, — — — — - slope – 2.9.

are associated mainly with large strain (large Λ_1) rather than largest enstrophy, strong tendency of alignment between ω and the eigenvector λ_1 associated with the largest eigenvalue Λ_1 of the rate of strain tensor s_{ij}, and fairly large curvature of vorticity lines. These regions, obviously, are dynamically most active, strongly non-Gaussian and possess structure.

$\mathcal{D}$. About a third of the volume in turbulent flows is occupied by regions with vortex *compressing, i.e.* negative enstrophy generation, large magnitude of Λ_3, alignment between ω and λ_3, large curvature of vortex lines, which to a large extent (but not only) is related to vortex *folding and tilting*. This part of turbulent flows plays an important role too, e.g. in generating a considerable part of the curvature of vortex lines along with the other part produced in the regions of largest enstrophy generation via self-induction.

It is noteworthy that the enstrophy dominated regions are well defined by, say, high enough enstrophy ω^2 and are tube/filament-like objects. They form a subset of much larger *locally* quasi-two-dimensional regions corresponding to large $\cos(\omega, \lambda_2)$, i.e alignment between ω and λ_2. However, it is not enough to specify the magnitude of the total strain s^2 in order to 'visualize' in a unique way the strain dominated regions, which – as demonstrated above – contain a number of *qualitatively* different regions. In particular, the set of strain dominated regions contains the dynamically significant subsets mentioned in $\mathcal{C}$. and $\mathcal{D}$. along with regions wrapped around the enstrophy dominated ones.

Most of the properties of the above mentioned three regions are not taken into account by models based on straight strained vortical objects.

The previous qualitative observations (mostly from DNS) on the '*little apparent structure in the low intensity component*' or the '*bulk of the volume*' with '*no particular visible structure*' should be interpreted that no *simple visible* structure has been observed so far in the bulk of the volume in the flow. It is a reflection of our inability to 'see' more intricate aspects of turbulence structure: intricacy and 'randomness' are not synonyms for the absence of structure. The same is true of all regions other than those with concentrated vorticity. However, structure is definitely present practically everywhere in turbulent flow, but it is more complex than just a collection of 'simple' objects such as vortex tubes, though the latter may only seem to be simple. Indeed, even in rather 'simple' nonturbulent configurations (*see*, e.g. Alekseenko and Shtork 1992; Fernandez, Zabusky and Gryanik 1995; Kida and Takaoka 1994; Kishiba, Okhitani and Kida 1994; Smith and Wei 1994 and references therein) the structure of the vorticity field is far more complicated than just a collection of simple objects such as vortex tubes, etc. This together with the limited role of the 'simple' objects in turbulent flows leads to the conclusion that it would be somewhat wishfully naive to expect that such a complicated phenomenon as turbulence can be described in terms of collections of such 'simple' (weakly interacting) objects *only*. On the contrary it seems that all the regions — concentrated vorticity, the background, regions of strong vorticity-strain (self) interaction and large enstrophy generation, and regions with negative enstrophy production and the rest — are in continuous interaction and mutual transformation and are strongly correlated due to the strong nonlocality of turbulence in physical space.

At this stage we are unable to find out how the structure of the regions differs from those with concentrated vorticity. One of the main issues for future research is to get more insight into this structure and in the dynamics of the interaction and mutual transformation of different regions/structures. This requires much more than the single point space-time statistics mostly used in this paper. In particular things such as time evolution, Lagrangian statistics (e.g. Huang 1996, Ruetsch and Maxey 1992, She *et al.* 1991, Yeung 1997 and references therein) relating the spatial structure and the time dimension have to be used and studied. Some such work is in progress.

Neither do we know whether our results will remain the same for large Reynolds numbers, since all the results shown above were obtained at a rather low Reynolds number ($Re_\lambda \approx 75$). However, a number of very similar results, such as various alignments, etc., were obtained in numerical simulations of the Euler equations (Bell and Marcus 1992, Brachet *et al.* 1992, Chorin 1994, Fernadez *et al.*1995, Ohkitani 1993, 1997, Ohkitani and Kishiba 1995, Porter *et al.* 1994, 1995, Pumir and Siggia 1990) thereby showing that - at least qualitatively - they should be valid at large Reynolds numbers.

Our hope is that the results of this paper are of importance for further basic research in turbulence and in their implications and consequences for a large number of existing and forthcoming theoretical descriptions of turbulence/turbulent flows, e.g. those based on expansions near a Gaussian field, and those representing the turbulent field as a collection of simple objects.

APPENDIX

Vortex stretching versus stretching of material lines (Tsinober 1995)

- - The equation for a material line element ℓ is a linear one and the vector ℓ is passive, *i.e.* the fluid flow doesn't 'know' anything whatsoever about ℓ. In other words the vector ℓ (as any passive vector) doesn't exert any influence on the fluid flow. The material element is stretched (compressed) locally at an exponential rate proportional to the rate of strain along the direction of ℓ, since the strain is independent of ℓ.
- - On the contrary the equation for vorticity is a nonlinear partial differential equation and the vector ω is an active one - it 'reacts back' on the fluid flow – vorticity and rate of strain tensor are composed of derivatives of the same velocity field, *i.e.* the strain does depend on ω and vice versa. Moreover this dependence is essentially nonlocal. Therefore, the rate of vorticity stretching/compressing is not exponential one and is unknown. There are much 'fever' vorticity lines than material ones - at each point there is typically only one vorex line, but infinitely many material lines, *i.e.* vortex lines are special (special material lines in case of inviscid flow). This leads to essential differences in the statistical properties of the two fields. In particular, positive correlation between curvature and rate of vortex stretching is a consequence of the active nature of vorticity and is a result of its self amplification process.
- - Consequently while a material element ℓ tends to be aligned with the largest (positive) eigenvector of s_{ij}, vorticity ω tends to be aligned with the intermediate (mostly positive) eigenvector of s_{ij}: the 'eigenframe' of s_{ij} rotates with an angular velocity $\mathbf{\Omega}_s$ of the order of vorticity ω (Dresselhaus and Tabor 1991). This

is mainly a consequence of the difference in the locality properties: stretching of material lines is a local process, whereas vortex stretching is essentially nonlocal.

- - For a Gaussian isotropic velocity field the enstrophy generation is identically zero, $\langle \omega_i \omega_j s_{ij} \rangle \equiv 0$ (Shtilman et al. 1993), whereas the mean rate of stretching of material lines is essentially positive (Batchelor 1952, Cocke 1961, Orszag 1977). The same is true of the mean rate of vortex stretching $\langle \omega_i \omega_j s_{ij} |\omega|^{-2} \rangle$ and for purely two-dimensional flows. This means that one can expect that in turbulent flows the mean growth rate of material lines is larger than for vorticity (Orszag 1977). Recently this was actually observed in decaying DNS turbulence (Huang 1996). Note that while in a Gaussian isotropic velocity field the volumes occupied by points with $\omega_i \omega_j s_{ij} > 0$ and $\omega_i \omega_j s_{ij} < 0$ are equal, in real turbulent flows the latter occupy about 1/3 of the flow volume. In other words the nature of vortex stretching process is dynamical and not a kinematic one.
- - An additional difference due to viscosity becomes essential for regions with concentrated vorticity, in which there is an approximate balance between enstrophy generation and its reduction. Vortex reconnection is allowed by nonzero viscosity. No such phenomena exist for material lines.

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